SAMPEL PERCUMA

# Latihan Mathematics T STPM Semester 1 Ikut Bab

## Set penuh: Kertas Percubaan STPM

Sem 1 · Mathematics T

**Sampel ini: 7 muka surat. Set penuh ada:**

- 3 fail PDF
- Semester 1: kertas percubaan (trial) negeri, subjek yang dipilih + skema
- Blueprint 30 hari Semester 1 (setiap subjek)
- Semester 3: percubaan 2024, 10 subjek, banyak negeri, soalan + skema dalam satu fail

# RM12.90

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Sampel bermula di muka surat sebelah

# Summative Test 1

**1** Given that $f(x) = 2 - \frac{1}{(x-1)^2}$ for $x < 1$. Find $k$ if $f^{-1}(k) = -2$

**2** The function $f$ is defined by $f: x \to x^3 - 1$, $x \in \mathbb{R}$.

(a) Show that f is a one-to-one function

(b) Find $f^{-1}$

(c) Verify that $f^{-1}(f(x)) = x$

**3** A function $f$ is given by $f: x \to 3x^2 + 2x - 2$

(a) If the domain of $f$ is the set of all real numbers, show that the function $f$ has no inverse.

(b) The function $f$ will have an inverse if the domain of $f$ is given by $D_f = \{x : x \leqslant k\}$.

(i) Find the maximum value of $k$

(ii) Find $f^{-1}$ and state its domain.

**4** If $g(x) = \frac{x+a}{bx+c}$ and $g^{-1}(x) = 1 + \frac{3}{x+1}$, find the values of the constants $a$, $b$ and $c$.

**5** If $f(x) = a + \frac{b}{x}$, $x \in \mathbb{R} \setminus \left\{0, a, \frac{-b}{a}\right\}$ and $f^{-1}(x) = f$, find a relation between $a$ and $b$. Hence find $f^{62}(x)$ in terms of $a$.

**6** If $f(x) = \sqrt{px+q}$ and graphs $y = f(x)$ and $y = f^{-1}(x)$ both pass through the point (3, 1), find the values of $p$ and $q$.

**7** The function $f$ is defined as $f(x) = \frac{3x+1}{x-c}$ for $x \neq c$ where c is a constant.

(a) Find an expression for $f^{-1}(x)$

(b) State the value of c if function $f$ is self-inverse

**8** The function $f$ is defined as $f: x \to 2e^x - 1$.

(a) State the domain for $f$.

(b) State the range for $f$.

(c) Show that $f^{-1}$ exists.

(d) Find $f^{-1}$

(e) Sketch the graphs of $y = f(x)$ and $y = f^{-1}(x)$ on the same axes.

**9** Functions f and g are defined by

$f: x \to \ln x$, $x > 0$ and $g: x \to 1 + |x|$, $x \in \mathbb{R}$.

(a) Find $(fog)(x)$

(b) Sketch the graph of $y = (fog)(x)$

(c) Use your graph to explain why $(fog)^{-1}$ does not exist.

(d) Given that $(fog)^{-1}$ exists when the domain for fog is change to $\{x : x \geqslant k\}$.

(i) State the minimum value for $k$

(ii) Find the value of $(fog)^{-1}(2)$

**10** Find the domain for each of the following functions:

(a) $f(x) = e^{2x+1}$

(b) $f(x) = \dfrac{1}{1-e^x}$

(c) $f(x) = \dfrac{1-x}{e^{2x}-3}$

**11** Find the domain for each of the following functions:

(a) $f(x) = 2\log(3+x)$

(b) $f(x) = \log x^2 + 3$

(c) $f(x) = 2\log x + 3$

(d) $f(x) = \sqrt{\log(x+3)}$

(a) $f(x) = \sqrt{(x+1)}\log(x+3)$

**12** Functions $f$ and $g$ each with domain $R$, are defined by

$f: x \to e^x$ and $g: x \to |x-1|$

(a) Find $(fog)(x)$.

(b) Sketch the graph of $y = (fog)(x)$.

(c) Use your graph to explain why $(fog)^{-1}$ does not exist.

(d) Given that $(fog)^{-1}$ exists when the domain for $fog$ is change to $\{x : x \leqslant k\}$.

(i) State the maximum value for $k$

(ii) Find the value of $(fog)^{-1}(2)$

**13** The function $f$ is defined as $f: x \to 2\ln x - 1$

(a) State the domain for $f$.

(b) State the range for $f$.

(c) Show that f is a one-to-one function

(d) Find $f^{-1}$

(e) Sketch the graphs of $y = f(x)$ and $y = f^{-1}(x)$ on the same axes.

**14** Given that $f: x \to \sqrt{\dfrac{1}{x}}$, $x > 0$

$g: x \to \dfrac{1}{3x-1}$, $x \neq \dfrac{1}{3}$ and

$h: x \to e^x - 1$, $x \in R$

find the composite function $fogoh$

**15** Find a polynomial function of degree 3 with $\frac{1}{2}$, $1-\sqrt{2}$ and $1+\sqrt{2}$ as zeros.

**16** Use the substitution $y = x + \dfrac{1}{x}$, or otherwise, solve the equation $x^4 - 4x^3 - 3x^2 - 4x + 1 = 0$, giving your answer in surd form

**17** The remainder when $3x^9 - ax + b$ is divided by $x - 1$ and $x + 1$ are equal

(a) Find $a$

(b) Show that $b$ can take any value

**18** The quadratic polynomial $p(x)$ has a factor $x + 1$, a remainder of 9 on division by $(x + 2)$ and a remainder of 6 when divided by $(x - 1)$. Find the polynomial $p(x)$ and solve the equation $p(x) = 0$.

# Summative Test 2

**1** Simplify (a) $\frac{(n+1)!}{(n-1)!}$ (b) $\frac{(3x+2)!}{(3n+4)!}$

**2** Use the Binomial Formula to expand $(p+q)^6$.

**3** Find the term that independent of $x$ in $\left(x^3+\frac{1}{x}\right)^{12}$.

**4** Given that $r$ is a positive integer and $f(r)=\frac{1}{r^2}$, find a single expression for $f(r)-f(r+1)$.

Hence, prove that

$$\sum_{r=1}^{4n}\left(\frac{2r+1}{r^2(r+1)^2}\right)=\frac{n(n+2)}{(n+1)^2}$$

**5** Use the standard results for $\sum_{r=1}^{n} r$ and $\sum_{r=1}^{n} r^2$, find the expansion in terms of $n$ for the sum of series

$$1^2+3^2+5^2+7^2+\ldots \text{ to } n \text{ terms.}$$

Simplify your answer as far as possible. Hence, find the value of $1^2+3^2+5^2+7^2+\ldots+59^2$ and deduce the value of

$$29^2+31^2+33^2+\ldots+59^2.$$

**6** Express $\frac{2}{4r^2-1}$ in partial fractions.

Show that $\sum_{r=1}^{n}\frac{2}{4r^2-1}=\frac{2n}{2n+1}$.

**7** Show that

$$\frac{1}{r(r+1)}-\frac{1}{(r+1)(r+2)}\equiv\frac{2}{r(r+1)(r+2)}$$

Hence, or otherwise find a simplified expression for

$$\sum_{r=1}^{n}\frac{1}{r(r+1)(r+2)}$$

**8** Express $\frac{1}{(2x-1)(2x+3)}$ in partial fractions

show that $\sum_{r=1}^{n}\frac{1}{(2r-1)(2r+3)}=\frac{1}{3}-\frac{n+1}{(2n+1)(2n+3)}$

and state whether or not the series is convergent.

**9** Find the coefficient of $x^2$ in the expansion of

$$\left(x+\frac{1}{x}\right)^5\left(x-\frac{1}{x}\right)^3.$$

**10** (a) Express $\frac{1}{(2n-1)(2n+1)}$ in partial fraction.

(b) Show that $\frac{1}{1\times3}+\frac{1}{3\times5}+\frac{1}{5\times7}+\ldots+\frac{1}{(2n-1)(2n+1)}$ is $\frac{1}{2}-\frac{1}{2(2n+1)}$.

(c) Find the sum of the series

$$\frac{1}{1\times3}+\frac{1}{3\times5}+\frac{1}{5\times7}+\ldots+\frac{1}{27\times29}.$$

**11** The first term of *A.P* is 7. The sum of the first six terms is 192, and the sum of the entire series is 4560. Determine the common difference, the number of terms and the last term.

**12** Find the sum of the infinite series

(a) $\sum_{n=0}^{\infty}\frac{3}{2n}$

(b) $1+(x-2)+(x-2)^2+(x-2)^3+\ldots$, given $|x-2|<1$

# Summative Test 3

**1** If $A = \begin{pmatrix} 2 & 5 \\ 1 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 2 & 4 \\ 3 & 1 \end{pmatrix}$, $C = \begin{pmatrix} 1 & 3 \\ 0 & 4 \end{pmatrix}$

Show that (a) $(A + B + C)^T = A^T + B^T = C^T$

(b) $(AB)^T = B^T A^T$

**2** Find $\begin{pmatrix} \frac{1}{9} & \frac{2}{9} \\ -\frac{2}{9} & \frac{5}{9} \end{pmatrix} \begin{pmatrix} 5 & -2 \\ 2 & 1 \end{pmatrix}$. Hence, solve the system of linear equations

$5x_1 - 2x_2 = 2$

$2x_1 + x_2 = 3$

**3** Solve $Ax = B$ if $A^{-1} = \begin{pmatrix} 1 & 1 & 1 \\ 2 & 4 & 9 \\ 5 & 3 & 0 \end{pmatrix}$ and $B = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}$

**4** Given that $A = \begin{pmatrix} 1 & 2 & x+1 \\ x & -3 & -4 \\ -1 & 2 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 2 & 1 & 4 \\ 0 & 1 & 1 \\ 0 & 0 & 3 \end{pmatrix}$ and $|A| = 12$, determine

(a) the values of $x$

(b) $|A^T|$

(c) $|A^2|$

(d) $|AB|$

**5** Show that $\begin{vmatrix} p & p+q & 1 \\ q & r+p & 1 \\ r & q+r & 1 \end{vmatrix} = (r-p)(2q-p-r)$ and deduce the value of $\begin{vmatrix} 4 & 7 & 1 \\ 3 & 6 & 1 \\ 2 & 5 & 1 \end{vmatrix}$

**6** Determine the values of $p$, $q$ and $r$ so that the matrix $\begin{pmatrix} q & 2-q & p+q+r \\ q^2 & p & p^2 \\ 4 & 4(p-1) & r \end{pmatrix}$ is a symmetric matrix.

**7** Find the inverse of $\begin{pmatrix} 2 & 2 & -1 \\ 0 & 4 & -1 \\ -1 & -2 & 1 \end{pmatrix}$ if it exists

**8** Use Gaussian-elimination method to show that the following system of linear equations is inconsistent

$x - 3y = 21$

$4x - 2y = 14$

$3x + 3y = 7$

**9** (a) Express the system of linear equations

$2x + 3y + z = 10$

$x - y + z = 4$

$4x - y - 5z = -8$

in the form $Ax = B$

(b) By using elementary row operations, find the inverse of $\begin{pmatrix} 2 & 3 & 1 \\ 1 & -1 & 1 \\ 4 & -1 & -5 \end{pmatrix}$.

(c) Hence, find the unique solutions for the system of linear equations given in (a).

# Summative Test 6

**1** $OABC$ is a parallelogram with $\overrightarrow{OA} = \underline{a}$ and $\overrightarrow{OC} = \underline{c}$. $P$ and $Q$ are the mid point of $OA$ and $OC$ respectively.

(a) Express the following in terms of $\underline{a}$ and $\underline{c}$.

(i) $\overrightarrow{BA}$ (ii) $\overrightarrow{CP}$ (iii) $\overrightarrow{QP}$

(b) If $OR = \frac{1}{2}\underline{a} + n\underline{c}$, where $n$ is a real number, find the locus of $R$.

**2** The coordinates of points $A$ and $B$ are respectively $(1, -2, 1)$ and $(0, 2, 3)$. If $R$ is on line $AB$ such that $AR : AB = 1 : 5$. Find $\overrightarrow{OR}$.

**3** The position vectors of points $A$ and $B$ with respect to the origin are $\underline{a} = 3\underline{j} - 5\underline{k}$ and $\underline{b} = 2\underline{i} + \underline{j} - \underline{k}$. Find

(a) $\overrightarrow{BA}$, (b) $|\overrightarrow{BA}|$,

(c) a unit vector in the direction of $\overrightarrow{AB}$.

**4** If $O$ is the origin and $|\overrightarrow{OA}| = 5$ and $\overrightarrow{OA}$ is in the direction $3\underline{i} - \underline{j} + \sqrt{6}\underline{k}$, find the coordinates of $A$.

**5** Given that $\underline{a} = 2\underline{i} + \underline{j} - \underline{k}$ and $\underline{b} = 3\underline{i} - 2\underline{j} - \underline{k}$, find the values of $p$ if $3p\underline{a}.2\underline{b} = p^2$.

**6** If $\underline{a} = \underline{i} + \underline{j} + \underline{k}$ and $\underline{b} = 3\underline{i} - 2\underline{j} + \underline{k}$, find the angle between

(a) $\underline{a}$ and $\underline{b}$, (b) $(\underline{a} + \underline{b})$ and $(\underline{a} - \underline{b})$.

**7** $ABC$ is a triangle. If $AB = 7$ cm, $AC = 2$ cm and $\overrightarrow{AB}.\overrightarrow{AC} = 5$, find

(a) the length of $BC$, (b) $\angle ABC$.

**8** Given that $\underline{u}$ and $\underline{v}$ and $\underline{w}$ are three non-zero vectors. If $(2\underline{u}.\underline{v})\underline{u} = 2\underline{v}$ and $(\underline{u}.3\underline{w})\underline{v} = \underline{w}$ find $|\underline{u}|$ and $|\underline{v}|$.

**9** Find the interior angles of the triangle $ABC$ whose vertices are $A(1, 2, 3)$, $B(-1, 2, 0)$ and $C(1, 2, -1)$.

**10** The angle between the vector $\alpha\underline{i} + 3\underline{j} + \underline{k}$ and the positive direction of the $x$-axis is 45°. Find the value of $\alpha$.

**11** Show that $\underline{u}$ and $\underline{v}$ are perpendicular if

(a) $\underline{u} = \underline{i} - \underline{j} + \underline{k}$ and $\underline{v} = 3\underline{i} - 2\underline{j} - 5\underline{k}$,

(b) $\underline{u} = |\underline{y}|\underline{x} + |\underline{x}|\underline{y}$ and $\underline{v} = |\underline{y}|\underline{x} - |\underline{x}|\underline{y}$ where $\underline{x}$ and $\underline{y}$ are two non-zero vectors.

**12** Given that $O$ is the origin. Coordinates of $A$, $B$ and $C$ are respectively $(2, 2, \alpha - 2)$, $(\alpha + 1, \alpha + 1, 3\alpha - 6)$ and $(3, 2, -5)$. Find $\alpha$ if

(a) $\overrightarrow{OA}$ is parallel to $\overrightarrow{AB}$, (b) $\overrightarrow{OC}$ is perpendicular to $\overrightarrow{AB}$.

**13** Given that $|\underline{a}| = 3$, $|\underline{b}| = 1$ and the angle between $\underline{a}$ and $\underline{b}$ is $\frac{\pi}{3}$ radian, find

(a) $\underline{a}.\underline{b}$, (b) $(2\underline{a} - \underline{b}).(\underline{a} + 2\underline{b})$, (c) $|3\underline{a} - \underline{b}|$.

**14** Vector $\underline{a}$, $\underline{b}$ and $\underline{c}$ are on the same plane. If $|\underline{b}| = 1$, $|\underline{c}| = 2$, $\underline{a}.\underline{b} = -2$, $\underline{b}.\underline{c} = 1$ and $\underline{c}.\underline{a} = 3$. Find $\underline{a}$ in terms of $\underline{b}$ and $\underline{c}$. (Hint : Let $\underline{a} = \alpha\underline{b} + \beta\underline{c}$)

# ANSWERS

## Chapter 1
### Summative Test 1

**1** $1\frac{8}{9}$

**2** $f^{-1} : x \to (x+1)^{\frac{1}{3}}, x \in R$

**3** (b) (i) $\frac{-1}{3}$

(ii) $f^{-1} : x \to \frac{-1}{3} + \sqrt{\frac{3x+7}{9}}, x \geqslant \frac{-7}{3}$

$Df^{-1} = \left[\frac{-7}{3}, +\infty\right)$

**4** $a = -4 \quad b = -1 \quad c = 1$

**5** $b = -a^2 ; f^{62}(x) = \frac{a^2}{a-x}$

**6** $p = -4 \quad q = 13$

**7** (a) $f^{-1}(x) = \frac{cx+1}{x-3}$

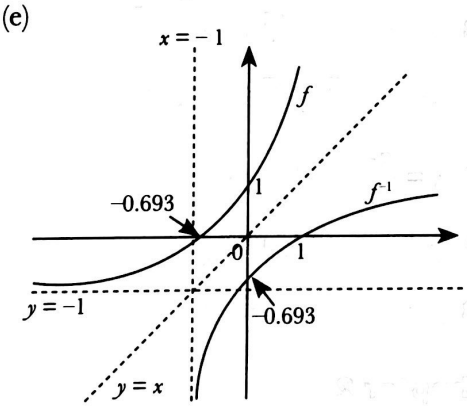
(b) $c = 3$

**8** (a) $D_f = (-\infty + \infty)$

(b) $R_f = (-1, +\infty)$

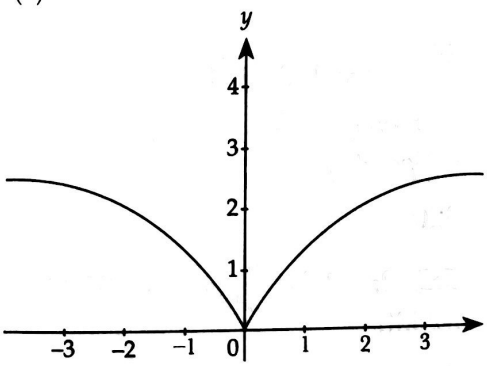
(d) $f^{-1}x \to ln\left(\frac{x+1}{2}\right), x > -1$

(e)




**9** (a) $(fog)(x) = ln(1 + |x|)$

(b)




(c) fog is not a one-one function

(d) (i) 0

(ii) $e^2 - 1$

**10** (a) $D_f = (-\infty, \infty)$

(b) $D_f = (-\infty, 0) \cup (0, \infty)$

(c) $D_f = \left(-\infty, \frac{1}{2}ln\,3\right) \cup \left(\frac{1}{2}ln\,3, \infty\right)$

**11** (a) $D_f = (-3, +\infty)$

(b) $D_f = (-\infty, 0) \cup (0, +\infty)$

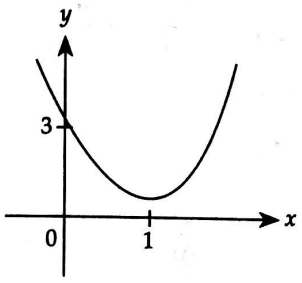
(c) $D_f = (0, +\infty)$

(d) $D_f = [-2, +\infty)$

(e) $D_f = [-1, +\infty)$

**12** (a) $(fog)(x) = e^{|x-1|}$

(b)




(d) (i) $k = 1$

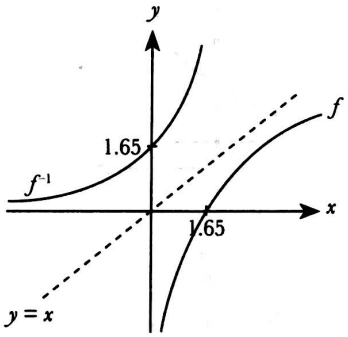
(ii) $(fog)^{-1}(2) = 1 - ln\,2$

**13** (a) $D_f = (0, +\infty)$

(b) $R_f = (-\infty, +\infty)$

(d) $f^{-1}(x) = e^{\frac{x+1}{2}}, x \in \mathbf{R}$

(e)




**14** $(fogoh) : x \to \sqrt{3e^x - 4}, x > ln\frac{4}{3}$

**15** $f(x) = x^3 - \frac{5}{2}x^2 + \frac{1}{2}$

**16** $x = \frac{5 \pm \sqrt{21}}{2}, \frac{1 \pm i\sqrt{3}}{2}$

**17** $a = -3$

**18** $p(x) = 4x^2 + 3x - 1$

$x = \frac{1}{4}$ or $x = -1$

**19** (a) $(-6, -4) \cup (2, 3)$

(b) $\left(-\infty, \frac{3}{5}\right] \cup [9, \infty) / \left\{\frac{3}{2}\right\}$

**20** $x \geqslant \frac{-3}{2}$ or $\left[\frac{-3}{2}, \infty\right)$

**21** $x = -3, -5$

**22** (a) $x < 3$ or $x > 7$, $(-\infty, 3)$ or $(7, \infty)$

(b) $0 \leqslant x \leqslant 6$, $[0, 6]$

(c) $-2 \leqslant x \leqslant -1$ or $x \geqslant 2$, $[-2, -1] \cup [2, \infty)$

**23** $x = \pm 9$

**24** $x < -3 \cup x \leqslant 4$

**25** $-1 < x \leqslant 2$

**26** $(-\infty, -2] \cup (1, 3]$

**27** $-1 < x < 1$

**28** $\frac{-5}{3} < x < 5$

**29** (a) $\frac{1}{16(x-3)} - \frac{1}{16(x+1)} - \frac{1}{4(x+1)^2}$

(b) $\frac{-5}{2(x+3)} + \frac{4}{x+2} - \frac{3}{2(x+1)}$

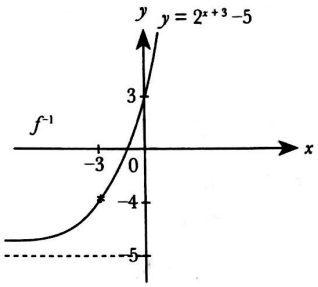
(c) $\frac{-1}{x+3} - \frac{2}{x-1} - \frac{8}{(x-1)^2}$

(d) $x - 1 + \frac{23}{5(x-3)} + \frac{27}{5(x+2)}$

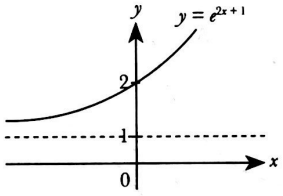
**30** $t = 2 \quad t = -1$

**31** $x = -9$ or $x = 1$

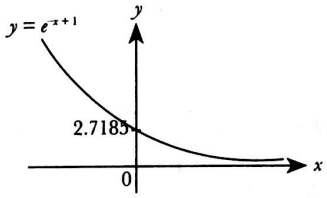
**32** (a)




(b)



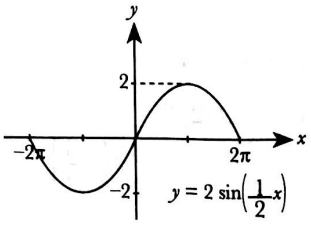

(c)



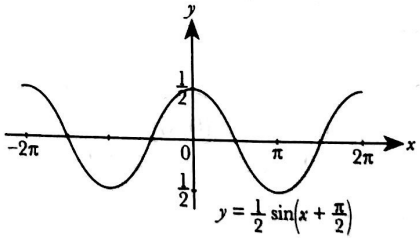

**33** $x = 1$ or $e^4/54.598$

**34** $x = 3$

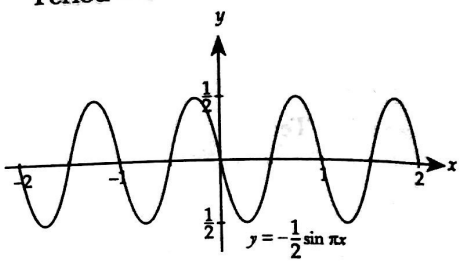
**36** (a)



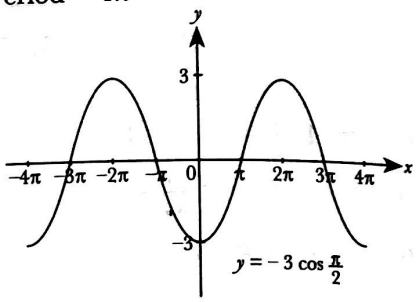

(b)




**37** (a) Amplitude $= \frac{1}{2}$
Period $= 1$




(b) Amplitude $= 3$
Period $= 4\pi$




**39** (a) $1 + \sqrt{5}$

**40** (a) Max value $= 2 + \sqrt{2}$ at $\frac{7\pi}{4}$

(b) Max value $= 8$ at $2.498$

**42** $\frac{-7}{25}$

**43** (a) $x = \frac{\pi}{3}, \frac{2\pi}{3}$ or $\frac{4\pi}{3}, \frac{5\pi}{3}$

(b) $x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

**44** $x = \frac{7\pi}{6}, \frac{11\pi}{6}$ or $\frac{\pi}{2}$

**47** $\frac{1}{x+1} + \frac{2}{x+2} + \frac{3}{(x+2)^2}$

**48** $p = 3, q = -1, r = -2$

## Chapter 2

### Summative Test 2

**1** (a) $n(n+1)$

(b) $\frac{1}{(3n+4)(3n+3)}$

**2** $p^6 + 6p^5q + 15p^4q^2 + 20p^3q^3 + 15p^2q^4 + 6pq^5 + q^6$

**3** 320

**5** $\sum_{r=1}^{n} (2r-1)^2 = \frac{n}{3}[4n^2 - 1]$, 35990, 32336

**7** $\frac{1}{4} - \frac{1}{2(n+1)(n+2)}$

**8** convergent

**9** $-6$

**10** $\frac{1}{2(2x-1)} - \frac{1}{2(2x+1)}, \frac{14}{29}$

TAMAT SAMPEL

# Suka sampel ini?

## Set penuh: Kertas Percubaan STPM

Sem 1 · Mathematics T

**Sampel ini: 7 muka surat. Set penuh ada:**

- 3 fail PDF
- Semester 1: kertas percubaan (trial) negeri, subjek yang dipilih + skema
- Blueprint 30 hari Semester 1 (setiap subjek)
- Semester 3: percubaan 2024, 10 subjek, banyak negeri, soalan + skema dalam satu fail

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