SAMPEL PERCUMA

Pra Pentaksiran STPM 2018 Mathematics T Semester 1 (Kulai, Johor)

Set penuh: Kertas Percubaan STPM

Sem 1 · Mathematics T

Sampel ini: 7 muka surat. Set penuh ada:

- 3 fail PDF
- Semester 1: kertas percubaan (trial) negeri, subjek yang dipilih + skema
- Blueprint 30 hari Semester 1 (setiap subjek)
- Semester 3: percubaan 2024, 10 subjek, banyak negeri, soalan + skema dalam satu fail

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Sampel bermula di muka surat sebelah

Section A [45 *marks*]

Answer **all** *questions in this section.*

1. Given the function $g(x) = \frac{1}{2x-5}$.

(a) Find the domain and range of $g(x)$. [2 marks]

(b) Show that $g(x)$ is a one-to-one function.

Hence, find $g^{-1}(x)$. [3 marks]

(c) On the same axis, sketch the graph of $g(x)$ and $g^{-1}(x)$. [3 marks]

(d) Show that $g \cdot g^{-1}(x) = x$. [2 marks]

2. (a) Verify the identity $\frac{2r-1}{r(r-1)} - \frac{2r+1}{r(r+1)} \equiv \frac{2}{(r-1)(r+1)}$. [2 marks]

(b) Hence, using the method of differences to prove that

$$\sum_{r=2}^{n} \frac{2}{(r-1)(r+1)} = \frac{3}{2} - \frac{2n+1}{n(n+1)}$$

[4 marks]

(c) Deduce the sum of the infinite series

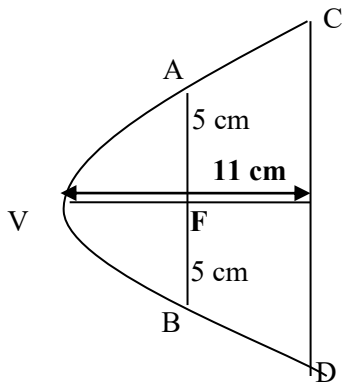
$\frac{1}{1\times3} + \frac{1}{2\times4} + \frac{1}{3\times5} + \cdots + \frac{1}{(n-1)(n+1)} + \cdots$ [2 marks]

3. Matrices **M** is given by $\mathbf{M} = \begin{pmatrix} 1 & 0 & 0 \\ 1 & -2 & 0 \\ -1 & 3 & 1 \end{pmatrix}$.

(a) Find the values of constants *a* and *b* if $\mathbf{M}^2 + a\mathbf{M} + b\mathbf{I} = \mathbf{0}$ where **I** is the 3×3 identity matrix. [4 marks]

(b) Hence, express $\mathbf{M}^3$ and $\mathbf{M}^{-1}$ in terms of **M** and **I**. [3 marks]

4. A cross-section of a parabolic reflector is shown in the figure below.



The bulb is located at the focus and the opening at the focus is 10 cm.

(a) Find an equation of the parabola. [3 marks]

(b) Find the diameter of the opening $|CD|$, 11 cm from the vertex. [2 marks]

5. Given $z_1 = 3 - 3i$ and $z_2 = 3 + 2i$.

(a) Write $\overline{z_1}$ in polar form. [3 marks]

(b) Express $\frac{\overline{z_1}z_2}{13} + \left[\frac{\overline{i^3}}{-z_2}\right]$ in the form of $a + bi,\ a, b \in R$. [3 marks]

6. The points A, B and C have position vectors, relative to the origin O, given by

$$\overrightarrow{OA} = \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}, \overrightarrow{OB} = \begin{pmatrix} 3 \\ 0 \\ 1 \end{pmatrix}, \overrightarrow{OC} = \begin{pmatrix} 1 \\ 1 \\ 4 \end{pmatrix}$$

The plane *m* is perpendicular to *AB* and contains the point *C*.

(a) Find the vector equation for the line passing through *A* and *B*. [2 marks]

(b) Obtain the equation of the plane *m*, giving your answer in the form $ax+by+cz=d$. [2 marks]

(c) The line through *A* and *B* intersects the plane *m* at point N.

Find position vector of N and show that CN=$\sqrt{13}$. [5 marks]

Section B [15 marks]

Answer **one** *question only in this section.*

You may answer all the questions **but** *only the first answer will be marked.*

7. (a) Sketch the graph of $y = |2\ \cos 2x|$ in the range of $0 \leq x \leq \pi$.

Hence, solve the inequality $|\cos 2x| > \frac{1}{2}$, where $0 \leq x \leq \pi$. [6 marks]

(b) (i) By first expanding $\cos\ (x + 45°)$, then express $\cos\ (x + 45°) - \sqrt{2}\ \sin x$ in the form $R\ \cos(x+\propto)$, where $R > 0$ and $0° < \propto < 90°$. Give the value of R correct to 4 significant figures and the value of $\propto$ correct to 2 decimal places. [5 marks]

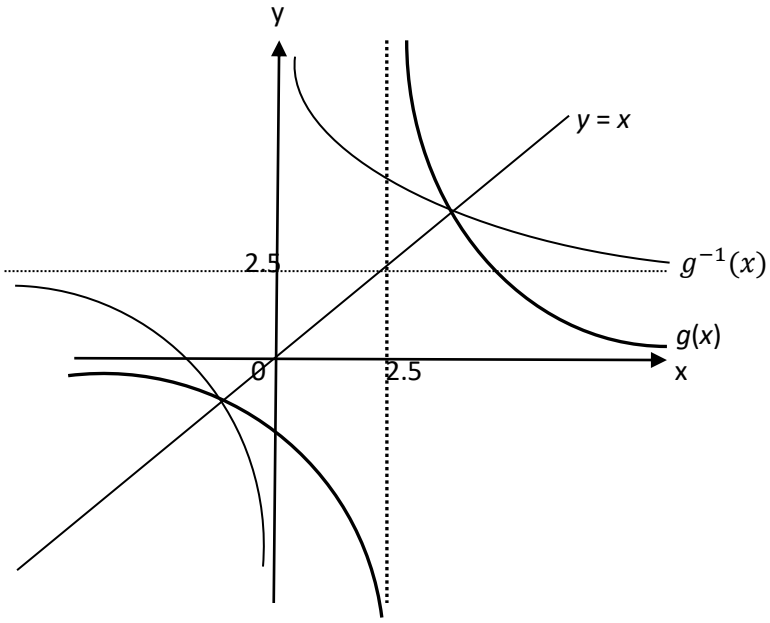
(ii) Hence, solve the equation $\cos\ (x + 45°) - \sqrt{2}\ \sin x = 2$, for $0° < x < 360°$. [4 marks]

8. Two companies A and B decided to award prizes to their employees for three work ethical values, namely punctuality (x), creativity (y) and efficiency (z). Company A decided to award a total of RM 3850 for the three values to 6, 2 and 3 employees respectively, while company B decided to award RM 3200 for the three values to 4, 1 and 5 employees respectively. The total amount for all the three prizes is RM 1000.

(a) Construct a system of linear equations to represent the above situation. [3 marks]

(b) By forming an augmented matrix, solve this equation system using Gaussian Elimination method. [7 marks]

(c) With the same total amount of money spent by company A and B, is it possible for Company A to award 15 employees for their creativity instead of 2 employees? Give your reason. [5 marks]

1	(a)	$D_g = \left(-\infty, \frac{5}{2}\right) \cup \left(\frac{5}{2}, \infty\right)$ $R_g = (-\infty, 0) \cup (0, \infty)$	B1 B1
	(b)	Since there is only one intersection point between the horizontal line and the graph, the function is one-to-one function. $\frac{1}{2y-5} = x$ $x(2y-5) = 1$ $y = \frac{1+5x}{2x}$ $g^{-1}(x) = \frac{1+5x}{2x}$	B 1 M 1 A1
	(c)		D 1- shape D 1- asymptote D 1 - all correct
	(d)	$g\,[g^{-1}(x)]$ $= g\left[\frac{1+5x}{2x}\right]$ $= \frac{1}{2\left[\frac{1+5x}{2x}\right] - 5}$ $= \frac{1}{\frac{1+5x-5x}{x}}$ $= x$	M 1 A1
		TOTAL	10 marks

2	(a)	$\frac{2r-1}{r(r-1)}-\frac{2r+1}{r(r+1)}$ $=\frac{(r+1)(2r-1)-(2r+1)(r-1)}{r(r-1)(r+1)}$ $=\frac{2r^2-r+2r-1-2r^2+2r-r+1}{r(r-1)(r+1)}$ $=\frac{2r}{r(r-1)(r+1)}$ $=\frac{2}{(r-1)(r+1)}$ [verified]	M 1 A 1
	(b)	$\sum_{r=2}^{n}\frac{2}{(r-1)(r+1)}$ $=\sum_{r=2}^{n}\left[\frac{2r-1}{r(r-1)}-\frac{2r+1}{r(r+1)}\right]$ $=\sum_{r=2}^{n}[f(r)-f(r+1)]$ $=f(2)-f(n+1)$ $=\frac{2(2)-1}{2(1)}-\frac{2(n+1)-1}{(n+1)(n+1-1)}$ $=\frac{3}{2}-\frac{2n+1}{n(n+1)}$	M 1 M1 M 1 (sub value) A 1
	(c)	$\frac{1}{2}\sum_{r=2}^{\infty}\frac{2}{(r-1)(r+1)}$ $=\frac{1}{2}\lim_{n\to\infty}\left[\frac{3}{2}-\frac{2n+1}{n(n+1)}\right]$ $=\frac{1}{2}\lim_{n\to\infty}\left[\frac{3}{2}-\frac{\frac{2}{n}+\frac{1}{n^2}}{1+\frac{1}{n}}\right]$ $=\frac{1}{2}\left(\frac{3}{2}-0\right)=\frac{3}{4}$	M 1 A 1
		TOTAL	8 MARKS

5	(a)	$\bar{z_1} = 3 + 3i$ $\lvert\bar{z_1}\rvert = \sqrt{3^2 + 3^2} = 3\sqrt{2}$ Argument of $\bar{z_1} = \tan^{-1}\left(\frac{3}{3}\right) = 0.785\ rad$ $\bar{z_1}\ in\ polar\ form$ $\bar{z_1} = 3\sqrt{2}\ (\cos 0.785 + i\ \sin 0.785)$	M1 M1 A1
	(b)	$\frac{\bar{z_1}z_2}{13} + \left[\frac{\bar{\imath}^3}{-z_2}\right] = \frac{(3+3i)(3+2i)}{13} + \left[\frac{\overline{-\imath}}{-(3+2\imath)}\right]$ $= \frac{3+15i}{13} + \left[\frac{\overline{\imath(3-2\imath)}}{(3+2\imath)(3-2\imath)}\right]$ $= \frac{3+15i}{13} + \left[\frac{\overline{2+3\imath}}{13}\right]$ $= \frac{3+15i+2-3i}{13}$ $= \frac{5+12i}{13}$	 M1 M1 A1
		TOTAL	6 MARKS
6	(a)	$\overrightarrow{AB} = \begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix}$ The line is parallel to $\overrightarrow{AB}$ The vector equation of line is $\boldsymbol{r} = \boldsymbol{i} + 2\boldsymbol{j} + \lambda(2\boldsymbol{i} - 2\boldsymbol{j} + \boldsymbol{k})$ Or $\boldsymbol{r} = 3\boldsymbol{i} + \boldsymbol{k} + \mu(2\boldsymbol{i} - 2\boldsymbol{j} + \boldsymbol{k})$	M1 A1
	(b)	$\boldsymbol{r} \cdot \boldsymbol{n} = \boldsymbol{c} \cdot \boldsymbol{n}$ $\boldsymbol{r} \cdot \begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 4 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix}$ = 2-2+4 = 4 Equation of plane is $2x - 2y + z = 4$	M1 A1
	(c)	Equation line: $\boldsymbol{r} = \boldsymbol{i} + 2\boldsymbol{j} + \lambda(2\boldsymbol{i} - 2\boldsymbol{j} + \boldsymbol{k})$ ---------- ① Equation plane: $2x - 2y + z = 4$ ---------- ② Substitute ① into ②, $2(1+ 2\lambda)$-$2(2-2\ \lambda) + \lambda = 4$ $9\ \lambda = 6$ $\lambda = \frac{2}{3}$ Position vector of N is $\frac{7}{3}\boldsymbol{i} + \frac{2}{3}\boldsymbol{j} + \frac{2}{3}\boldsymbol{k}$ $\overrightarrow{OC} = \begin{pmatrix} 1 \\ 1 \\ 4 \end{pmatrix}, \overrightarrow{ON} = \begin{pmatrix} \frac{7}{3} \\ \frac{2}{3} \\ \frac{2}{3} \end{pmatrix}$ then $\overrightarrow{CN} = \begin{pmatrix} \frac{4}{3} \\ -\frac{1}{3} \\ -\frac{10}{3} \end{pmatrix}$ $\lvert\overrightarrow{CN}\rvert = \sqrt{\left(\frac{4}{3}\right)^2 + \left(-\frac{1}{3}\right)^2 + \left(-\frac{10}{3}\right)^2}$	 M1 A1 A1 M1 A1

		Basic angle = 26.56° $\left(x + 71.57^\circ\right) = 333.44^\circ, 386.56^\circ$ $x = 261.87^\circ, 314.99^\circ$	M 1 M 1 A 1
		TOTAL	15 MARKS
8	(a)	$6x + 2y + 3z = 3850$ $4x + y + 5z = 3200$ $x + y + z = 1000$	B 1 B 1 B 1
	(b)	$PX = Q$ $\begin{pmatrix} 6 & 2 & 3 \\ 4 & 1 & 5 \\ 1 & 1 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3850 \\ 3200 \\ 1000 \end{pmatrix}$	B 1
		$\left(\begin{array}{ccc\|c} 6 & 2 & 3 & 3850 \\ 4 & 1 & 5 & 3200 \\ 1 & 1 & 1 & 1000 \end{array}\right)$	B 1
		$\xrightarrow{R_1 \leftrightarrow R_3}\left(\begin{array}{ccc\|c} 1 & 1 & 1 & 1000 \\ 4 & 1 & 5 & 3200 \\ 6 & 2 & 3 & 3850 \end{array}\right)$	M 1
		$\xrightarrow{4R_1 - R_2 \to R_2}\left(\begin{array}{ccc\|c} 1 & 1 & 1 & 1000 \\ 0 & 3 & -1 & 800 \\ 6 & 2 & 3 & 3850 \end{array}\right)$	
		$\xrightarrow{6R_1 - R_3 \to R_3}\left(\begin{array}{ccc\|c} 1 & 1 & 1 & 1000 \\ 0 & 3 & -1 & 800 \\ 0 & 4 & 3 & 2150 \end{array}\right)$	M 1
		$\xrightarrow{4R_2 - 3R_3 \to R_3}\left(\begin{array}{ccc\|c} 1 & 1 & 1 & 1000 \\ 0 & 3 & -1 & 800 \\ 0 & 0 & -13 & -3250 \end{array}\right)$	
		$-13z = -3250$ $z = 250,$ $3y - (250) = 800$ $y = 350,$ $x + 350 + 250 = 1000$ $x = 400$ $x = 400,\ y = 350,\ z = 250$	M 1 A 1 A 1(all)
	(c)	$6x + 15y + 3z = 3850$ $4x + y + 5z = 3200$ $x + y + z = 1000$	B 1
		$Let\ P = \begin{pmatrix} 6 & 15 & 3 \\ 4 & 1 & 5 \\ 1 & 1 & 1 \end{pmatrix}$	M 1
		$\lvert P\rvert = 6\begin{vmatrix} 1 & 5 \\ 1 & 1 \end{vmatrix} - 15\begin{vmatrix} 4 & 5 \\ 1 & 1 \end{vmatrix} + 3\begin{vmatrix} 4 & 1 \\ 1 & 1 \end{vmatrix}$ $= 6(-4) - 15(-1) + 3(3) = 0$	M 1 A 1
		Since $\lvert P\rvert = 0$, P is singular , thus the system of linear equation have no unique solution. It is not possible for company A to award 15 employees for their creativity instead of 2 with the same total amount of money.	A 1
		TOTAL	15 MARKS

TAMAT SAMPEL

Suka sampel ini?

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