SAMPEL PERCUMA

Soalan Latihan Mathematics M STPM Semester 1 dengan Skema

Set penuh: Kertas Percubaan STPM

Sem 1 · Mathematics M

Sampel ini: 8 muka surat. Set penuh ada:

- 3 fail PDF
- Semester 1: kertas percubaan (trial) negeri, subjek yang dipilih + skema
- Blueprint 30 hari Semester 1 (setiap subjek)
- Semester 3: percubaan 2024, 10 subjek, banyak negeri, soalan + skema dalam satu fail

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Sampel bermula di muka surat sebelah

TEST 2

Section A [45 *marks*]

*Answer **all** questions in this section.*

1. A piecewise function f is defined by

$$\mathrm{f}: x \to \begin{cases} 5 - 2x - x^2 & , x < 0 \\ \dfrac{5}{x+1} & , x \geq 0 \end{cases}$$

(a) Sketch the graph of f, and find its range, [5 *marks*]

(b) Explain why f^{-1} does not exist. [2 *marks*]

2. The fifth term and the eighth term of a geometric series are $\frac{3}{4}$ and $\frac{3}{32}$ respectively.

(a) Find the sum of the first n terms, S_n, of this series. [4 *marks*]

(b) Hence, show that $\sum_{n=1}^{N} S_n = 24N - S_N$. [4 *marks*]

3. Given matrix $\mathrm{M} = \begin{pmatrix} 3 & m(m-3) & m^2 \\ -6 & 1 & 4m \\ 3 & -2 & 4 \end{pmatrix}$.

Use the properties of determinants,

(a) explain why if $m = 2$ or -2, then $|\mathrm{M}| = 0$. [3 *marks*]

(b) determine $|\mathrm{M}|$ when $m = 0$. [2 *marks*]

4. The curve C has the equation $16y^3 + 9x^2y - 54x = 0$.

Express $\frac{dy}{dx}$ in terms of x and y. [3 *marks*]

Hence, find the coordinates of the points where $\frac{dy}{dx} = 0$. [6 *marks*]

5. Express $\frac{4x^2 - 13x + 15}{(x-1)^2(4-x)}$ in partial fractions. [4 *marks*]

Hence find the exact value of $\int_2^3 \frac{4x^2 - 13x + 15}{(x-1)^2(4-x)}\,dx$. [3 *marks*]

6. The variables x and y, where $x, y > 0$ are related by the differential equation

$$2xy\frac{dy}{dx} = y^2 - 4x^2.$$

By Using the substitution $y = ux$, show that the differential equation may be transformed to

$$x\frac{du}{dx} = -\frac{4 + u^2}{2u}.$$ [3 *marks*]

Solve this differential equation, and hence, find y, in terms of x with the condition that $y = 1$ when $x = 1$. [6 *marks*]

Section B [15 marks]
Answer only *one* question in this section.

7. Expand $\frac{1}{\left(1+\frac{x}{n}\right)^n}$, $n \in \mathrm{Z}^+$, in ascending powers of x up to and including the term in x^3.[4 *marks*]

Given that the coefficient of x^3 is $-\frac{10}{27}$, determine the value of n.

[3 *marks*]

With this value of n, expand $\frac{\sqrt{1-x}}{\left(1+\frac{x}{n}\right)^n}$ in ascending powers of x up to and including the term in x^3.

[4 *marks*]

Hence, by taking suitable value of x, estimate $\sqrt{10}$ correct to three decimal places.[4 *marks*]

8. Given that $y = \frac{3x}{1-x^2}$.

(a) Write down the equations of the asymptotes of the graph. [2 *marks*]

(b) Find $\frac{dy}{dx}$ and show that $\frac{dy}{dx} > 0$ for all real values of x. [4 *marks*]

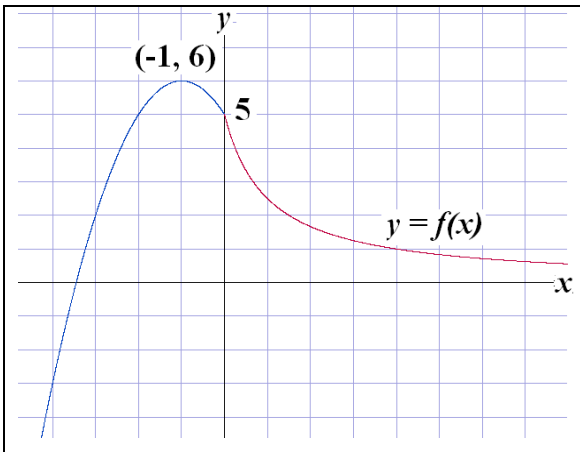
(c) Sketch the graph of y and state the intervals where the graph is concave upwards. [6 *marks*]

(d) By sketching a suitable curve on the same diagram, determine the number of real roots of the equation

$$(1 - x^2)e^{-x} - 3x = 0$$ [3 *marks*]

SKEMA TEST 2

1

	**D1** **D1**	**D1: Correct shape for x <0** **or x > 0** **D1: All correct with labelling,** **(-1, 6), and 5**
For x<0: $f(x) = 6 - (x + 1)^2$	**B1**	
$f(-1) = 6$, $f(0) = 5$	**M1**	
$\therefore$ range = $(-\infty, 6]$	**A1**	**[5]**

A horizontal line $y = k$, $0 < k < 6$, cuts the curve at two points, $\therefore$ f is not one-to one function $\therefore$ f^{-1} does not exist	**M1** **A1**	 **[2]**

2

5^{th} term, $ar^4 = \frac{3}{4}$, 8^{th} term, $ar^7 = \frac{3}{32}$	**B1**	
$r^3 = \frac{4}{32} = \frac{1}{8} \Rightarrow r = \frac{1}{2}$, $a = 12$ $S_n = \frac{12\left(1-\left(\frac{1}{2}\right)^n\right)}{1-\frac{1}{2}} = 24\left(1-\left(\frac{1}{2}\right)^n\right)$	**M1** **M1** **A1**	
$\sum_{n=1}^{N} S_n = 24\sum_{n=1}^{N}\left(1-\left(\frac{1}{2}\right)^n\right) = 24N - 24\sum_{n=1}^{N}\left(\frac{1}{2}\right)^n$ $= 24N - 24\frac{\frac{1}{2}\left(1-\left(\frac{1}{2}\right)^N\right)}{1-\left(\frac{1}{2}\right)}$ $= 24N - 24\left(1-\left(\frac{1}{2}\right)^N\right) = 24N - S_N$	**M1** **M1** **A1** **A1**	

3	If $m = 2$, M = $\begin{pmatrix} 3 & -2 & 4 \\ -6 & 1 & 8 \\ 3 & -2 & 4 \end{pmatrix}$ then 1st row and 3rd row are identical $\Rightarrow$ |M| = 0	**M1**	
	If $m = -2$, M = $\begin{pmatrix} 3 & 10 & 4 \\ -6 & 1 & -8 \\ 3 & -2 & 4 \end{pmatrix}$ then 1st column is 3/4 the 3rd column $\Rightarrow$ |M| = 0	**M1A1**	
	When $m = 0$, M is a lower triangular matrix $\Rightarrow$ |M| = 3 x 1 x 4 = 12	**M1A1**	

4	$16y^3 + 9x^2y - 54x = 0$ $48y^2\frac{dy}{dx} + 18xy + 9x^2\frac{dy}{dx} - 54 = 0$	**M1A1**	
	$3(16y^2 + 3x^2)\frac{dy}{dx} = 18(3 - xy)$ $\frac{dy}{dx} = \frac{6(3 - xy)}{(16y^2 + 3x^2)}$	**A1**	
	When $\frac{dy}{dx} = 0$, $3 - xy = 0 \Rightarrow y = \frac{3}{x}$ or $x = \frac{3}{y}$	**B1**	
	$16y^3 + 9\left(\frac{3}{y}\right)^2 y - 54\left(\frac{3}{y}\right) = 0$ $\Rightarrow 16y^4 - 81 = 0$ $\Rightarrow (4y^2 - 9)(4y^2 + 9) = 0$ $\Rightarrow (2y - 3)(2y + 3)(4y^2 + 9) = 0$	**M1** **M1A1**	
	$\Rightarrow y = \frac{3}{2}, \quad -\frac{3}{2}$ $y = \frac{3}{2}, \quad x = 2$ or $y = -\frac{3}{2}, \quad x = -2$	**M1**	
	$\therefore$ the points C are $\left(2, \frac{3}{2}\right)$ and $\left(-2, -\frac{3}{2}\right)$	**A1**	

TEST 4

Section A [45 *marks*]

*Answer **ALL** questions in this section.*

1 Sketch, on the same axes, the graphs of $y = |3x - 2|$ and $y = 2 - x^2$. Hence, solve the inequality $|3x - 2| \geq 2 - x^2$. *[6]*

2 Given that $(x + 2)$ is a factor of the polynomial $p(x) = 2x^3 - x^2 + ax - 6$. Determine the value of a. *[4]*

Factorise $p(x)$ completely.

Hence, determine the set of values of x for which $\frac{p(x)}{1-x} \geq 0$. *[2]*

3 Express $\frac{1+2x}{\sqrt{9+4x}}$ in ascending powers of x up to the terms in x^3. Determine the set of values of x for which the expansion is valid. *[4]*

By substituting a suitable value of x into your expansion, obtain an approximate value of $\sqrt{\frac{4}{11}}$ correct to three decimal places.

[5]

4 Given that $xy + 2y = 2e^x \ln x$,

(a) Show that $(x^2 + 2x)\frac{dy}{dx} = 2e^x + x^2 y + xy$. *[4]*

(b) Find the gradient of the curve at the point when $x = 1$. *[4]*

5 (a) Given that $A = \begin{pmatrix} 2 & 3 & -1 \\ 6 & -3 & 1 \\ 4 & -2 & -2 \end{pmatrix}$. Find the inverse of matrix A, using Gauss-Jordan elimination method. *[5]*

(b) Hence, solve the system of linear equations:

$$\begin{aligned} 4x + 6y - 2z &= 6 \\ 6x - 3y + z &= 5 \\ 2x - y - z &= 2 \end{aligned}$$. *[4]*

6 (a) Express in partial fractions $\frac{2x^2 - 6x - 1}{(2x^2 + 1)(2x - 3)}$. *[3]*

(b) Hence, evaluate the value of $\int_2^3 \frac{4x^2 - 12x - 2}{3(2x^2 + 1)(2x - 3)} dx$ *[4]*

Section B

*Answer **any ONE** question in this section.*

7 (a) Express in partial fractions $\frac{1}{r^3 + 3r^2 + 2r}$. *[4]*

(b) Using the results obtained, deduce the sum to *n* terms of the series

$$\frac{1}{1 \cdot 2 \cdot 3} + \frac{1}{2 \cdot 3 \cdot 4} + \frac{1}{3 \cdot 4 \cdot 5} + ... \quad \textit{[6]}$$

(c) Explain why the series is a convergent. *[2]*

(d) Deduce the sum of the series

$$\frac{1}{10 \cdot 11 \cdot 12} + \frac{1}{11 \cdot 12 \cdot 13} + \frac{1}{12 \cdot 13 \cdot 14} + ... + \frac{1}{20 \cdot 21 \cdot 22} \quad \textit{[3]}$$

8 (a) Find the integral value of $\int_1^2 x \ln x dx$ *[4]*

(b) The curve $y = x^2 - x \ln x$, $x > 0$ has a point of inflexion at point P.

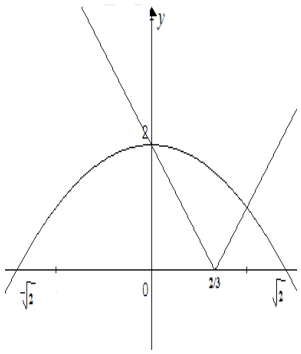
(i) Find the coordinates of point *P*. *[4]*

(ii) Determine the range of values of *x* where the curve concaves upward. *[2]*

(iii) Sketch the graphs of $y = x^2 - x \ln x$ and $y = x - 1$ on the same axes a calculate the area bounded by the two curves, $x = 1$ and $x = 2$. *[5]*

SKEMA TEST 4

Marking Schemes

No	Answers	Marks
1		G2 Intercept 1
	$3x-2=2-x^2$	B1
	$x=1$	
	$\therefore x\le 0, x\ge 1$	A2
2	$p(-2)=-16-4-2a-6=0$	M1
	$a=-13$	A1
	$p(x)=(x+2)(2x^2-5x-3)$	M1
	$=(x+2)(2x+1)(x-3)$	A1
	$\left\{x:-2\le x-\frac{1}{2},1<x\le 3\right\}$	M1A1
3	$\frac{1+2x}{\sqrt{9+4x}}=\frac{1+2x}{3}(1+\frac{4}{9}x)^{-\frac{1}{2}}$	B1
	$=\frac{1}{3}(1+2x)\left[1+\left(-\frac{1}{2}\right)\left(\frac{4}{9}x\right)+\frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{2!}\left(\frac{4}{9}x\right)^2+\frac{\left(-\frac{1}{2}x\frac{3}{2}x\frac{5}{2}\right)}{3!}\left(\frac{4}{9}x\right)^3+...\right]$	M1
	$=\frac{1}{3}(1+2x)\left[1-\frac{2}{9}x+\frac{2}{27}x^2-\frac{20}{729}x^3+.....\right]$	
	$=\frac{1}{3}\left[1-\frac{2}{9}x+\frac{2}{27}x^2-\frac{20}{729}x^3+...+2x-\frac{4}{9}x^2+\frac{4}{27}x^3+...\right]$	M1
	$=\left[\frac{1}{3}+\frac{16}{27}x-\frac{10}{81}x^2-\frac{88}{2187}x^3+.....\right]$	A1
	Valid for $.-\frac{9}{4}<x<\frac{9}{4}$	B1

	Substitute $x = 0.5$, $\dfrac{1+2x}{\sqrt{9+4x}} = \sqrt{\dfrac{4}{11}}$	B1
	$\therefore \sqrt{\dfrac{4}{11}} \approx \left[\dfrac{1}{3} + \dfrac{16}{27}\times 0.5 - \dfrac{10}{81}\times 0.5^2 - \dfrac{88}{2187}\times 0.5^3\right]$	
	$= 0.604$ (to 3dp)	M1 A1
4a)	$x\dfrac{dy}{dx} + y + 2\dfrac{dy}{dx} = \dfrac{2e^x}{x} + 2e^x$	M1A1
	$x\dfrac{dy}{dx} + y + 2\dfrac{dy}{dx} = \dfrac{2e^x}{x} + xy + 2y$	M1
	$x^2\dfrac{dy}{dx} + 2x\dfrac{dy}{dx} = 2e^x + x^2y + xy$	
	$(x^2 + 2x)\dfrac{dy}{dx} = 2e^x + x^2y + xy$	A1
b)	$x = 1,\ y + 2y = 2e\ln 1$	M1 A1
	$y = 0$	
	$(1+2)\dfrac{dy}{dx} = 2e$	M1
	$\dfrac{dy}{dx} = \dfrac{2}{3}e$	A1
5a)	$(A \mid I) = \begin{pmatrix} 2 & 3 & -1 & 1 & 0 & 0 \\ 6 & -3 & 1 & 0 & 1 & 0 \\ 4 & -2 & -2 & 0 & 0 & 1 \end{pmatrix}$	B1
	$\rightarrow \begin{pmatrix} a & b & e & h & k & l \\ 0 & c & f & m & n & p \\ 0 & d & g & q & r & s \end{pmatrix}$	M1A1
	$\rightarrow \begin{pmatrix} a & 0 & e & h & k & l \\ 0 & c & f & m & n & p \\ 0 & 0 & g & q & r & s \end{pmatrix}$	M1
	$\rightarrow \begin{pmatrix} 1 & 0 & 0 & \frac{1}{8} & \frac{1}{8} & 0 \\ 0 & 1 & 0 & \frac{1}{4} & 0 & -\frac{1}{8} \\ 0 & 0 & 1 & 0 & \frac{1}{4} & -\frac{3}{8} \end{pmatrix}$	M1
	$A^{-1} = \begin{pmatrix} \frac{1}{8} & \frac{1}{8} & 0 \\ \frac{1}{4} & 0 & -\frac{1}{8} \\ 0 & \frac{1}{4} & -\frac{3}{8} \end{pmatrix}$	A1

TAMAT SAMPEL

Suka sampel ini?

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- 3 fail PDF
- Semester 1: kertas percubaan (trial) negeri, subjek yang dipilih + skema
- Blueprint 30 hari Semester 1 (setiap subjek)
- Semester 3: percubaan 2024, 10 subjek, banyak negeri, soalan + skema dalam satu fail

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