CONFIDENTIAL*

954/1 **STPM 2014**

MATHEMATICS T
PAPER 1

One and a half hours

PEPERIKSAAN PERCUBAAN PENGGAL 1
SIJIL TINGGI PERSEKOLAHAN MALAYSIA 2014

SMK XXXXXXXXXXXXXXXXXXXX
2013

Instruction to candidates:

DO NOT OPEN THIS QUESTION PAPER UNTIL YOU ARE TOLD TO DO SO.

Answer **all** *questions in Section A and any* **one** *question in Section B.*

Answers may be written in either English or Bahasa Malaysia.

All necessary working should be shown clearly.

Scientific calculators may be used. Programmable and graphic display calculators are prohibited.

A list of mathematical formulae is provided on page of this question paper.

This question paper consists of 5 printed pages.

MATHEMATICAL FORMULAE

Trigonometrical identities

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B + \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 + \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Sums of series

For an arithmetic series

$$S_n = \tfrac{1}{2}n(a + \ell) = \tfrac{1}{2}n[2a + (n-1)d]$$

For a geometric series

$$S_n = \frac{a(1 - r^n)}{1 - r}, \quad a \neq 1$$

Binomial expansions

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \cdots + \binom{n}{r}a^{n-r}b^r + \cdots + b^n, \quad n \in \mathbb{Z}^+$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \cdots + \frac{n(n-1)\cdots(n-r+1)}{r!}x^r + \cdots, \quad n \in \mathbb{Q}, |x| < 1$$

Conics

Parabola with vertex (h, k), focus $(a + h, k)$ and directrix $x = -a + h$

$$(y-k)^2 = 4a(x-h)$$

Ellipse with centre (h, k) and foci $(-c + h, k)$, $(c + h, k)$, where $c^2 = a^2 - b^2$

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

Hyperbola with centre (h, k) and foci $(-c + h, k)$, $(c + h, k)$, where $c^2 = a^2 + b^2$

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$

Section A [45 marks]
Answer all questions in this section

1. The functions f and g are defined by :

$$\text{f} : x \mapsto x + 4, \qquad x \in \Re,\ a \le x \le b,$$
$$\text{g} : x \mapsto x^2 - 4x, \qquad x \in \Re,\ |x| \le 4$$

(a) By finding the values of $g(1)$ and $g(3)$, explain why the inverse function $\text{g}^{-1}(x)$ is not defined [2 *marks*]

(b) Given that the function $g \circ f(x)$ is defined, find the minimum value of a and the maximum value of b. [3 marks]

2.(a) Evaluate $\sum_{r=1}^{100} \left(3^r - 3^{r-1}\right)$, give your answer in the standard form $A \times 10^n$. [3 marks]

(b) Express $\dfrac{4}{(2-x)\sqrt{1+3x}}$ in the form of increasing power of x including the term of x^3.

Determine the range of values of x so that this expansion is valid. [6 marks]

3. Matrix P is invertible if $|\text{P}| \neq 0$ where $|\text{P}|$ is the determinant of P.

The matrix $A = \begin{bmatrix} 3 & 4 & -1 \\ 1 & 0 & 3 \\ 2 & 5 & -4 \end{bmatrix}$ has an inverse A^{-1} because $\begin{vmatrix} 3 & 4 & -1 \\ 1 & 0 & 3 \\ 2 & 5 & -4 \end{vmatrix} = -10$.

(a) Find A^{-1} by using the method of elementary row operations. [3 marks]

(b) Solve the following system of linear equations by method of matrices that involves A and A^{-1}.

$$3x + 4y = z + 1$$
$$2x + 5y - 4z = 3$$
$$x + 3z = 2$$

[4 marks]

(c) State the value of

(i) $\begin{vmatrix} 6 & 4 & -1 \\ 2 & 0 & 3 \\ 4 & 5 & -4 \end{vmatrix}$ (ii) $\begin{vmatrix} 3 & 4 & -1 \\ 2 & 5 & -4 \\ 1 & 0 & 3 \end{vmatrix}$ [2 marks]

4.(a) Given $p(1 + 5i) – 2q = 3 + 7i$, find the values of p and q if p and q are both real numbers. [3 marks]

(b) Express the complex number $\sqrt{3}+i$ in the form $r(\cos\theta+i\sin\theta)$, where r is the modulus and θ is the argument of the complex number.
Hence, simplify $(\sqrt{3}+i)^5$. [5 marks]

5. Show that the two curves $4x^2 + 9y^2 = 36$ and $4x^2 – y^2 = 4$ have the same foci.

For the hyperbola, state the equations of the asymptotes. [6 marks]

Sketch the curves $4x^2 + 9y^2 = 36$ and $4x^2 – y^2 = 4$ on the same axes, showing clearly the asymptotes of the hyperbola. [4 marks]

6. The points A, B and C have position vectors $\underline{a}=\underline{i}+\underline{j}+2\underline{k}$, $\underline{b}=3\underline{i}+2\underline{j}+4\underline{k}$ and $\underline{c}=-\underline{i}+4\underline{j}-4\underline{k}$ respectively.

Find : (a) $\underline{a}\bullet\underline{b}$ [1 mark]

(b) $(\underline{b}-\underline{a})\times(\underline{c}-\underline{a})$ [3 marks]

Section B [15 marks]
Answer any **one** question in this section

7.(a) The polynomial Q(x) is defined by Q(x) = x^3 + mx^2 + 5x – n .
If Q(x) has a quadratic factor (x^2 + 5) and a zero of –2 , find the values of m and n.
[3 marks]

(b) Express $\cos x + 2\sin x$ in the form $R\sin(x+\alpha)$ where R is positive and $0 < \alpha < \frac{\pi}{2}$.

Hence, state the maximum value of $\cos x + 2\sin x$ as well as the corresponding value of x in the range $0 < x < \frac{\pi}{2}$ correct to two decimal places. [5 *marks*]

(c) Prove that $\cot\theta - \tan\theta = 2\cot 2\theta$. [3 *marks*]

Hence or otherwise, solve the equation $\cot\theta - \tan\theta = \sqrt{12}$ for $0^\circ \le \theta \le 360^\circ$. [4 *marks*]

8. The line l has equation $\mathbf{r} = \begin{pmatrix} 5 \\ 0 \\ 5 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}$, $\lambda \in \Re$.

(a) Show that l lies in the plane whose equation is $\mathbf{r} \cdot \begin{pmatrix} -1 \\ 2 \\ 0 \end{pmatrix} = -5$. [3 marks]

(b) Find the position vector of A, the foot of the perpendicular from the origin O to l. [4 marks]

(c) Find an equation of the plane containing O and l. [4 marks]

(d) Find the position vector of the point P where l meets the plane π whose equation is $\mathbf{r} \cdot \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix} = 11$. [4 marks]

1. The functions f and g are defined by :

$\text{f} : x \mapsto x + 4, \qquad x \in \Re\ ,\ a \le x \le b\ ,$

$\text{g} : x \mapsto x^2 - 4x, \qquad x \in \Re\ ,\ |x| \le 4$

(a) By finding the values of $g(1)$ and $g(3)$, explain why the inverse function $\text{g}^{-1}(x)$ is not defined [2 *marks*]

(b) Given that the function $g \circ f(x)$ is defined, find the minimum value of *a* and the maximum value of *b*. [3 marks]

1.(a)	$g(1) = -3 \quad , \quad g(3) = -3$ $g(1) = g(3) = -3$ g is not one-to-one $\therefore$ function $g^{-1}(x)$ not defined	M1 A1	Get $g(1)$ and $g(3)$ and conclude
1.(b)	For function $g \circ f(x)$ defined $R_f \subset D_g$ $f(x) \in D_g \quad x \in D_f$ $f(x) \in [-4\ ,\ 4]$ $-4 \le f(x) \le 4$ $-4 \le x + 4 \le 4$ $-8 \le x \le 0$ But $a \le x \le b$ $\therefore$ minimum $a = -8$, maximum $b = 0$	M1 A1 A1	

2.(a) Evaluate $\sum_{r=1}^{100} \left(3^r - 3^{r-1}\right)$, give your answer in the standard form $A \times 10^n$. [3 marks]

(b) Express $\dfrac{4}{(2-x)\sqrt{1+3x}}$ in the form of increasing power of *x* including the term of x^3.

Determine the range of values of *x* so that this expansion is valid. [6 marks]

2(a)	$\sum_{r=1}^{100} (3^r - 3^{r-1}) = (3^1 - 3^0) + (3^2 - 3^1) + (3^3 - 3^2) + \ldots + (3^{99} - 3^{98}) + (3^{100} - 3^{99})$ $= 3^{100} - 3^0$ $= 3^{100} - 1$ $= 5.15 \times 10^{47}$	M1 A1 A1	
(b)	$\dfrac{4}{(2-x)\sqrt{1+3x}} = 2\left(1 - \dfrac{x}{2}\right)^{-1}(1+3x)^{-\frac{1}{2}}$ $= 2\left[1 + (-1)(-\dfrac{x}{2}) + \dfrac{(-1)(-2)}{2!}(-\dfrac{x}{2})^2 + \dfrac{(-1)(-2)(-3)}{3!}(-\dfrac{x}{2})^3 + \ldots.\right]$ $\left[1 + (-\dfrac{1}{2})(3x) + \dfrac{(-\frac{1}{2})(-\frac{3}{2})}{2!}(3x)^2 + \dfrac{(-\frac{1}{2})(-\frac{3}{2})(-\frac{5}{2})}{3!}(3x)^3 + \ldots\right]$	M1 M1	See $k(1-\frac{x}{2})^{-1}(1+3x)^{-\frac{1}{2}}$ Either one expanded

	$= 2\left[1+\frac{1}{2}x+\frac{1}{4}x^2+\frac{1}{8}x^3+...\right]\left[1-\frac{3}{2}x+\frac{27}{8}x^2-\frac{135}{16}x^3+...\right]$ $= 2(1-x+\frac{23}{8}x^2-7x^3+...)$ $= 2-2x+\frac{23}{4}x^2-14x^3+...$	A1 A1	correctly Either one series correct
	The expansion is valid when $\left\|-\frac{x}{2}\right\|<1$ and $\|3x\|<1$ $\|x\|<2$ and $\|x\|<\frac{1}{3}$ $\therefore \|x\|<\frac{1}{3}$ or $-\frac{1}{3}<x<\frac{1}{3}$	M1 A1	

3. Matrix P is invertible if $|\mathrm{P}| \neq 0$ where $|\mathrm{P}|$ is the determinant of P.

The matrix $A=\begin{bmatrix}3 & 4 & -1\\ 1 & 0 & 3\\ 2 & 5 & -4\end{bmatrix}$ has an inverse A^{-1} because $\begin{vmatrix}3 & 4 & -1\\ 1 & 0 & 3\\ 2 & 5 & -4\end{vmatrix}=-10$.

(a) Find A^{-1} by using the method of elementary row operations. [3 marks]

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[4 marks]

(c) State the value of

(i) $\begin{vmatrix}6 & 4 & -1\\ 2 & 0 & 3\\ 4 & 5 & -4\end{vmatrix}$ (ii) $\begin{vmatrix}3 & 4 & -1\\ 2 & 5 & -4\\ 1 & 0 & 3\end{vmatrix}$ [2 marks]

3(a)	$[A\|I] = \begin{bmatrix}3 & 4 & -1 & 1 & 0 & 0\\ 1 & 0 & 3 & 0 & 1 & 0\\ 2 & 5 & -4 & 0 & 0 & 1\end{bmatrix}$ $= \begin{bmatrix}1 & 0 & 3 & 0 & 1 & 0\\ 3 & 4 & -1 & 1 & 0 & 0\\ 2 & 5 & -4 & 0 & 0 & 1\end{bmatrix}$ $R_1 \leftrightarrow R_2$ $= \begin{bmatrix}1 & 0 & 3 & 0 & 1 & 0\\ 0 & 4 & -10 & 1 & -3 & 0\\ 0 & 5 & -10 & 0 & -2 & 1\end{bmatrix}$, $-3R_1+R_2 \rightarrow R_2$, $-2R_1+R_3 \rightarrow R_3$ $\vdots$	M1 M1 A1	Idea from $[A\|I]$ to get $[I\|A^{-1}]$ See two ERO carried out correctly

Suka sampel ini?

Kertas Percubaan STPM

Sem 1 · Mathematics T

- Semester 1: kertas percubaan (trial) negeri, subjek yang dipilih + skema
- Blueprint 30 hari Semester 1 (setiap subjek)
- Semester 3: percubaan 2024, 10 subjek, banyak negeri, soalan + skema dalam satu fail

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