SAMPEL PERCUMA

Trial STPM 2015 Mathematics M Semester 1 Pulau Pinang

Set penuh: Kertas Percubaan STPM

Sem 1 · Mathematics M

Sampel ini: 6 muka surat. Set penuh ada:

- 3 fail PDF
- Semester 1: kertas percubaan (trial) negeri, subjek yang dipilih + skema
- Blueprint 30 hari Semester 1 (setiap subjek)
- Semester 3: percubaan 2024, 10 subjek, banyak negeri, soalan + skema dalam satu fail

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Sampel bermula di muka surat sebelah

TEST 1

Section A [45 marks]

Answer **all** *questions in this section.*

1. The function f is defined by

$$f : x \to e^x, \quad x \geq 0$$

(a) State the range of f. [1 mark]

(b) Find f^{-1}. [2 marks]

(c) Sketch the graph of f and f^{-1}. [3 marks]

2. (a) Show that $\frac{1}{r^2} - \frac{1}{(r+1)^2} = \frac{2r+1}{r^2(r+1)^2}$ [2 marks]

(b) Hence, find the sum of the first n terms of the series

$$\frac{3}{1^2 \times 2^2} + \frac{5}{2^2 \times 3^2} + \frac{7}{3^2 \times 4^2} + ...$$ [4 marks]

3. A system of linear equations is given by

$$x + y - z = -2$$
$$2x - y + z = 5$$
$$-x + 2y + \lambda z = 1$$

where λ are real number. Show that the augmented matrix for the system may be reduced to

$$\left(\begin{array}{ccc|c} 1 & 1 & -1 & -2 \\ 0 & -3 & 3 & 9 \\ 0 & 0 & \lambda+2 & 8 \end{array}\right)$$

Hence, determine the values of λ and k so that the system of linear equations has no solution. [6 marks]

4. Given that $f(x) = x^3 + px^2 + x + q$ where p and q are constant, is divisible by (x -1). When divided by (x – 2), the remainder is 17.

(a) Find the value of p and q. [4 marks]

(b) Show that $f(x) = 0$ has only one real root. [3 marks]

(c) Express $\frac{11x+9}{f(x)}$ as partial fraction. [4 marks]

5. (a) Given that $x^2 + 2xy - y^2 = 1$, show that $\frac{dy}{dx} = \frac{x+y}{y-x}$. [3 marks]

(b) Determine the value of $\int_0^1 xe^{-2x}dx$ [5 marks]

6. The variable x and y are related by the differential equation $\frac{dy}{dx} = \frac{y+2x}{x}$ for $x > 0$.

Find the particular solution if $y = 2$ when $x = 1$. [8 marks]

Section B [15 marks]
Answer any **one** *questions in this section.*

7. a) Given $A = \begin{pmatrix} t-2 & 4 & 3 \\ 1 & t+1 & -2 \\ 0 & 0 & t-4 \end{pmatrix}$, find the value of t if A is a singular matrix.

[5 marks]

b) It is given that $P = \begin{pmatrix} 2 & -1 & 1 \\ 1 & -1 & -1 \\ 2 & -2 & -1 \end{pmatrix}$.

i) Find P^{-1} using elementary row operation. [6 marks]

ii) Hence, solve the following system of linear equations:

$$2x - y + z = 12$$
$$x - y - z = 1$$
$$2x - 2y - z = 6$$

[4 marks]

8. Given a curve $y = \dfrac{x^2 + 4}{x^2 - 4}$,

(a) State the asymptotes of the curve. [2 marks]

(b) Find the coordinates of the turning point of the curve. [6 marks]

(c) Sketch the curve. [3 marks]

(d) Find the set of values for k where equation $x^2 + 4 = k(x^2 - 4)$ has

(i) no real root

(ii) one real root,

(ii) two real and distinct roots.

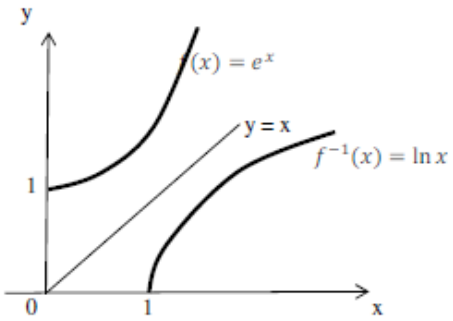
END OF QUESTION PAPER

SKEMA TEST 1
SMK Bukit Mertajam (Trial STPM)
Marking Scheme Mathematics M Paper 1 2015

1. a) $R_f = \{y : y \geq 1\}$ A1

b) Let $f^{-1} = y$
$f(y) = x$
$e^y = x$ M1
$y = \ln x$
$f^{-1}(x) = \ln x, x \geq 1$
$f^{-1} : x \to \ln x, x \geq 1$ A1

c)



$f(x)$	D1
$f^{-1}(x)$	D1
All correct	D1

2.

a $\frac{1}{r^2} - \frac{1}{(r+1)^2} = \frac{(r+1)^2 - r^2}{r^2(r+1)^2}$ B1
$= \frac{r^2 + 2r + 1 - r^2}{r^2(r+1)^2}$
$= \frac{2r+1}{r^2(r+1)^2}$ A1

b $\frac{3}{1^2 \times 2^2}, \frac{5}{2^2 \times 3^2}, \frac{7}{3^2 \times 4} \ldots\ldots$ B1
$u_r = \frac{2r+1}{r^2(r+1)^2}$
$\frac{3}{1^2 \times 2^2} + \frac{5}{2^2 \times 3^2} + \frac{7}{3^2 \times 4} + \ldots\ldots$
$= \sum_{r=1}^{n} \frac{2r+1}{r^2(r+1)^2}$ M1
$= \sum_{r=1}^{n} \frac{1}{r^2} - \frac{1}{(r+1)^2} = \left[\frac{1}{1} - \frac{1}{4} + \right.$
$\frac{1}{4} - \frac{1}{9} +$
$\frac{1}{9} - \frac{1}{16} +$
$\vdots$
$\left.\frac{1}{n^2} - \frac{1}{(n+1)^2} + \right.$ M1

$= \quad 1 - \frac{1}{(n+1)^2}$ A1

3. $\left(\begin{array}{ccc|c} 1 & 1 & -1 & -2 \\ 2 & -1 & 1 & 5 \\ -1 & 2 & \lambda & 1 \end{array}\right)$ B1

$= \left(\begin{array}{ccc|c} 1 & 1 & -1 & -2 \\ 0 & -3 & 3 & 9 \\ -1 & 2 & \lambda & 1 \end{array}\right)$ $R_2 = R_2 - 2R_1$ M1

$= \left(\begin{array}{ccc|c} 1 & 1 & -1 & -2 \\ 0 & -3 & 3 & 9 \\ 0 & 3 & \lambda - 1 & -1 \end{array}\right)$ $R_3 = R_1 + R_3$ M1

$= \left(\begin{array}{ccc|c} 1 & 1 & -1 & -2 \\ 0 & -3 & 3 & 9 \\ 0 & 0 & \lambda + 2 & 8 \end{array}\right)$ $R_3 = R_2 + R_3$ M1

For no solution $\lambda + 2 = 0$ M1

$\lambda = -2$ A1

4. a) $f(1) = 1 + p + 1 + q = 0$

$f(2) = 2^3 + 2^2 p + 2 + q + 17$ either one correct M1

$p + q = -2$.....(1)

$4p + q = 7$.....(2) Both (1) & (2) correct M1

(2) − (1) $3p = 9$ Solving (1) & (2) M1

$p = 3$

$q = -5$ both p & q correct A1

b) $f(x) = 0$

$x^3 + 3x^2 + x - 5 = 0$

$(x-1)(x^2 + 4x + 5) = 0$ M1

$(x^2 + 4x + 5) = (x+2)^2 + 1 > 0 \quad \forall x \in \Re$ M1

$\therefore \quad x = 1$ (only real root) A1

c) Let $\frac{11x+9}{f(x)} = \frac{11x+9}{(x-1)(x^2+4x+5)}$

$= \frac{A}{(x-1)} + \frac{Bx+C}{x^2+4x+5}$

$A(x^2 + 4x + 5) + (Bx + C)(x - 1) = 11x + 9$ M1

When $x = 1$: $10A = 20 \quad \Rightarrow \quad A = 2$

When $x = 0$: $5A - C = 9 \quad \Rightarrow \quad C = 1$

Comparing coefficient of x^2: $A + B = 0 \quad \Rightarrow \quad B = -2$

any two correct A1, all correct A2

$\frac{11x+9}{f(x)} = \frac{2}{(x-1)} + \frac{1-2x}{(x^2+4x+5)}$ A1

5. a) $x^2 + 2xy - y^2 = 1$

$2x + 2x\left(\frac{dy}{dx}\right) + y(2) - 2y\left(\frac{dy}{dx}\right) = 0$ M1

$\frac{dy}{dx}(2x - 2y) = -2x - 2y$ M1

$\frac{dy}{dx} = \frac{-2x - 2y}{2x - 2y}$

$= \frac{x + y}{y - x}$ A1

b) Let

$u = x \qquad \frac{dv}{dx} = e^{-2x}$

$\frac{du}{dx} = 1 \qquad v = \frac{e^{-2x}}{-2}$

$\int_0^1 xe^{-x}dx$

$= \left[x\left(\frac{e^{-2x}}{-2}\right)\right]_0^1 - \int_0^1 -\frac{1}{2}e^{-2x}dx$ M1 (without limit)

$= \left[-\frac{xe^{-2x}}{2}\right]_0^1 - \left[-\frac{1}{2}\left(\frac{e^{-2x}}{-2}\right)\right]_0^1$ M1 (without limit)

$= \left[-\frac{xe^{-2x}}{2} - \frac{1}{4}e^{-2x}\right]_0^1$ M1

$= \left(-\frac{1}{2}e^{-2} - \frac{1}{4}e^{-2}\right) - \left[0 - (\frac{1}{4})\right]$ M1

$= \frac{1}{4} - \frac{3}{4}e^{-2}$

$= \frac{1}{4}(1 - 3e^{-2})$ A1

7. a) $\det A = (t-4)\begin{vmatrix} t-2 & 4 \\ 1 & t+1 \end{vmatrix}$ M1

$= (t-4)\left[(t-2)(t+1)-4\right]$ M1

$= (t-4)(t^2-t-6)$ M1

$= (t-4)(t-3)(t+2)$ M1

$t = 4, t = 3, t = -2$ A1

b) i) $\left(\begin{array}{ccc|ccc} 2 & -1 & 1 & 1 & 0 & 0 \\ 1 & -1 & -1 & 0 & 1 & 0 \\ 2 & -2 & -1 & 0 & 0 & 1 \end{array}\right)$ B1

$\left(\begin{array}{ccc|ccc} 1 & -1 & -1 & 0 & 1 & 0 \\ 2 & -1 & 1 & 1 & 0 & 0 \\ 2 & -2 & -1 & 0 & 0 & 1 \end{array}\right) R_1 \leftrightarrow R_2$

$\left(\begin{array}{ccc|ccc} 1 & -1 & -1 & 0 & 1 & 0 \\ 0 & 1 & 3 & 1 & -2 & 0 \\ 2 & -2 & -1 & 0 & 0 & 1 \end{array}\right) R_2 \to R_2 - 2R_1$

$\left(\begin{array}{ccc|ccc} 1 & -1 & -1 & 0 & 1 & 0 \\ 0 & 1 & 3 & 1 & -2 & 0 \\ 0 & 0 & 1 & 0 & -2 & 1 \end{array}\right) R_3 \to R_3 - 2R_1$

$\left(\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & -1 & 0 \\ 0 & 1 & 3 & 1 & -2 & 0 \\ 0 & 0 & 1 & 0 & -2 & 1 \end{array}\right) R_1 \to R_1 + R_2$

$\left(\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & -1 & 0 \\ 0 & 1 & 0 & 1 & 4 & -3 \\ 0 & 0 & 1 & 0 & -2 & 1 \end{array}\right) R_2 \to R_2 - 3R_3$ M3

$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 3 & -2 \\ 0 & 1 & 0 & 1 & 4 & -3 \\ 0 & 0 & 1 & 0 & -2 & 1 \end{array}\right) R_1 \to R_1 - 2R_3$ A1

$P^{-1} = \begin{pmatrix} 1 & 3 & -2 \\ 1 & 4 & -3 \\ 0 & -2 & 1 \end{pmatrix}$ A1

ii) $\begin{pmatrix} 2 & -1 & 1 \\ 1 & -1 & -1 \\ 2 & -2 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 12 \\ 1 \\ 6 \end{pmatrix}$ B1

$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 & 3 & -2 \\ 1 & 4 & -3 \\ 0 & -2 & 1 \end{pmatrix}\begin{pmatrix} 12 \\ 1 \\ 6 \end{pmatrix}$ M1

$= \begin{pmatrix} 3 \\ -2 \\ 4 \end{pmatrix}$ A1

$x = 3, y = -2, z = 4$ A1

TAMAT SAMPEL

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