SAMPEL PERCUMA

Trial STPM 2015 Mathematics M Semester 1 Seremban

Set penuh: Kertas Percubaan STPM

Sem 1 · Mathematics M

Sampel ini: 7 muka surat. Set penuh ada:

- 3 fail PDF
- Semester 1: kertas percubaan (trial) negeri, subjek yang dipilih + skema
- Blueprint 30 hari Semester 1 (setiap subjek)
- Semester 3: percubaan 2024, 10 subjek, banyak negeri, soalan + skema dalam satu fail

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Sampel bermula di muka surat sebelah

Name : …….. Class : ……..

950/1 STPM 2015

PEPERIKSAAN PERCUBAAN

PENGGAL 1

MATHEMATICS M

PAPER 1

(One and a half hours)

Instructions to candidates:

DO NOT OPEN THIS QUESTION PAPER UNTIL YOU ARE TOLD TO DO SO.

Answer **all** *questions in Section A and any* **one** *question in Section B.*

All necessary working should be shown clearly.

Scientific calculators may be used. Programmable and graphic display calculators are prohibited.

A list of mathematical formulae is provided on page 3 *of this question paper.*

This question paper consists of 3 printed pages.

Section A

[45 *marks*]

Answer **all** *questions in this section.*

1. A polynomial leaves a remainder of 6 when divided by $(x - 2)$ and can be divided exactly by $(x + 2)$. Find the remainder when the polynomial is divided by $(x^2 - 4)$. [5 marks]

2. *(a)* Expand $\left(8+\frac{4}{x}\right)^{\frac{1}{3}}$ up to and including the term in $\left(\frac{1}{x}\right)^3$ and determine the range of values of x for which the expansion is valid. [5 marks]

(b) Hence, by substituting a suitable value of x, find the value of $\sqrt[3]{65}$ correct to four decimal places. [3 marks]

3. Find the complete solution of the following system of equations using Gaussian elimination.

$$\begin{aligned} -3x - 5y + 36z &= 10 \\ -x \quad\quad + \quad 7z &= 5 \\ x + \ y - 10z &= -4 \end{aligned}$$

[8 marks]

4. *(a)* Given that $f(x) = \ln\left(\frac{1-x}{x^2+3}\right)$, $x < 1$, find $f'(x)$ [3 marks]

(b) Hence, find the range of values of x for which the function is decreasing. [5 marks]

5. *(a)* On the same axes, sketch the graph of $y = e^x$ and $y = e^{-x}$. [3 marks]

(b) Hence, calculate the area of the region bounded by these two curves and the line $y = 3$. [4 marks]

6. The variables x and y, where $x, y > 0$ are related by the differential equation

$$(x^2 - 4)\frac{dy}{dx} = y$$

(a) Find the general solution of the differential equation [5 marks]

(b) Given that $y = 1$ when $x = 3$, show that the particular solution of the differential equation is given by $y^4 = \left|\frac{5(x-2)}{x+2}\right|$. [3 marks]

(c) Hence, find the values of x when $y = \frac{1}{2}$ [1 mark]

Section B

[15 *marks*]

Answer any **one** *question in this section.*

7. It is given that the curve $y = \frac{\ln x}{x}$, where x > 0 has a stationary point.

(a) Find the stationary point of the curve and determine its nature. [6 marks]

(b) Determine the coordinates of the point of inflexion of the curve, and state the intervals for which the curve is concave upwards and concave downwards. [4 marks]

(c) Sketch the graph of $y = \frac{\ln x}{x}$, x > 0 .

Hence, show that $x^e \le e^x$, for all positive values of x [5 marks]

8. The functions f and g are defined as follows

$$f : x \to \ln(3x + 2)$$

$$g : x \to \frac{e^x - 2}{3}$$

(a) Show algebraically that f is a one-to-one function. [3 marks]

(b) Find the composite function $f \circ g$ and $g \circ f$.

Hence, state the relationship between the functions f and g [5 marks]

(c) On the same axes, sketch the graphs of f and g and state the domain and range of f and g. [7 marks]

MATHEMATICAL FORMULAE

Binomial Expansion:

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \cdots + \binom{n}{r}a^{n-r}b^r + \cdots + b^n \text{, where } n \in Z^+$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \cdots + \frac{n(n-1)\cdots(n-r+1)}{r!}x^r + \cdots \text{, where } n \in Q,\ |x| < 1$$

Integration:

$$\int \frac{f'(x)}{f(x)}dx = \ln|f(x)|$$

$$\int u\frac{dv}{dx}dx = uv - \int v\frac{du}{dx}dx$$

END OF QUESTION PAPER

SCHEME MATEHAMTICS M TRIAL TERM 1- 2015
Section A

1. A polynomial leaves a remainder of 6 when divided by $(x - 2)$ and can be divided exactly by $(x + 2)$. Find the remainder when the polynomial is divided by $(x^2 - 4)$. [5 marks]

Scheme

(Since divisor is a quadratic expression, the remainder is a linear function)

Let the remainder be $Ax + B$

Given P(2) = 6; $2A + B = 6$ ……(1) B1

P(-2) = 0; $-2A + B = 0$ ……(2) B1

(1) + (2) $2B = 6$

$B = 3$

Substitute B = 3 in (1) $2A = 3$ M1 Solving

$A = \frac{3}{2}$ A1 (both values)

Remainder is $\frac{3}{2}x + 3$ A1

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2. (a) Expand $\left(8 + \frac{4}{x}\right)^{\frac{1}{3}}$ up to and including the term in $\left(\frac{1}{x}\right)^3$ and determine the range of values of x for which the expansion is valid. [5 marks]

(b) Hence, by substituting a suitable value of x, find the value of $\sqrt[3]{65}$ correct to four decimal places. [3 marks]

Scheme

$$\left(8 + \frac{4}{x}\right)^{\frac{1}{3}} = 8^{\frac{1}{3}}\left(1 + \frac{1}{2x}\right)^{\frac{1}{3}}$$ B1

$$= 2\left(1 + \frac{1}{3}\left(\frac{1}{2x}\right) + \frac{\frac{1}{3}\left(-\frac{2}{3}\right)}{2.1}\left(\frac{1}{2x}\right)^2 + \frac{\frac{1}{3}\left(-\frac{2}{3}\right)\left(-\frac{5}{3}\right)}{3.2.1}\left(\frac{1}{2x}\right)^3 + \ldots\right)$$ M1

$$= 2\left(1 + \frac{1}{6x} - \frac{1}{36x^2} + \frac{5}{648x^3} - \ldots\right)$$

$$= 2 + \frac{1}{3x} - \frac{1}{18x^2} + \frac{5}{324x^3} - \ldots$$ A1

Expansion is valid for $\left|\frac{1}{2x}\right| < 1$

$\left|\frac{1}{x}\right| < 2$ M1

$-2 < \frac{1}{x} < 2$

$\therefore x < -\frac{1}{2}$ *or* $x > \frac{1}{2}$ A1

Substitute x = 32 B1

This question paper is CONFIDENTIAL until the examination is over. **CONFIDENTIAL**

4. (a) Given that $f(x)=\ln\left(\frac{1-x}{x^2+3}\right)$, $x<1$, find $f'(x)$ [3 marks]

(b) Hence, find the range of values of x for which the function is decreasing. [5 marks]

Scheme

$f(x)=\ln\left(\frac{1-x}{x^2+3}\right)$, $x<1$

$f(x)=\ln(1-x)-\ln(x^2+3)$ M1

$f'(x)=-\frac{1}{1-x}-\frac{2x}{x^2+3}$ M1

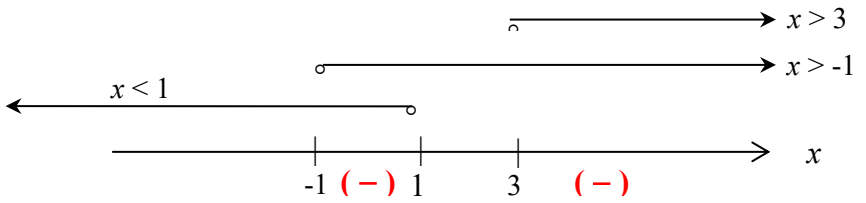
$=\frac{-1(x^2+3)}{1-x}-\frac{2x(1-x)}{x^2+3}$

$=\frac{x^2-2x-3}{(1-x)(x^2+3)}$

$=\frac{(x-3)(x+1)}{(1-x)(x^2+3)}$ A1

For f(x) decreasing f(x) < 0,

$\frac{(x-3)(x+1)}{(1-x)(x^2+3)}<0$ B1



M1(3 lines correctly drawn)

M1(identify correct interval)

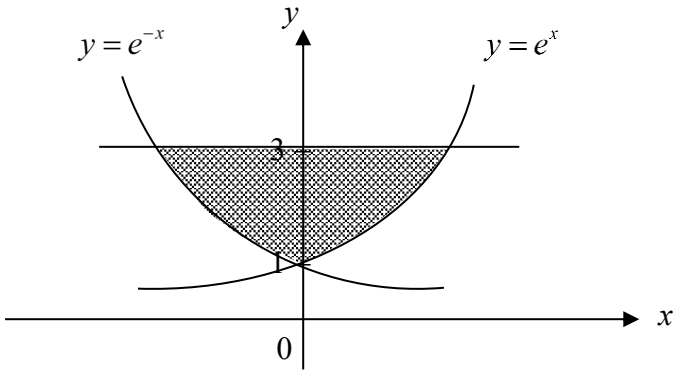
Since x < 1 , the range of value of x is $\{x: -1 < x < 1\}$ M1 A1

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5. (a) On the same axes, sketch the graph of $y=e^x$ and $y=e^{-x}$. [3 marks]

(b) Hence, calculate the area of the region bounded by these two curves and the line $y = 3$. [4 marks]

Scheme



D1 $y=e^x$

D1 $y=e^{-x}$

D1 all correct

Section B

7. It is given that the curve $y = \dfrac{\ln x}{x}$, where x > 0 has a stationary point.

   (a) Find the stationary point of the curve and determine its nature. [6 marks]

   (b) Determine the coordinates of the point of inflexion of the curve, and state the intervals for which the curve is concave upwards and concave downwards. [4 marks]

   (c) Sketch the graph of $y = \dfrac{\ln x}{x}$, x > 0 .

   Hence, show that $x^e \le e^x$, for all positive values of x [5 marks]

Scheme

(a) $y = \dfrac{1}{x}\ln x$

$\dfrac{dy}{dx} = -\dfrac{1}{x^2}\ln x + \dfrac{1}{x^2} = \dfrac{1}{x^2}(1-\ln x)$ M1

$\dfrac{dy}{dx} = 0 \quad \Rightarrow \dfrac{1}{x^2}(1-\ln x) = 0$ M1

$1 - \ln x = 0$

$\ln x = 1$

$x = \text{e}$

$\therefore \; y = \dfrac{1}{e}\ln e = \dfrac{1}{e}$

Stationary point = $\left(e, \dfrac{1}{e}\right)$ A1

$\dfrac{d^2y}{dx^2} = \dfrac{x^2\left(-\dfrac{1}{x}\right) - (1-\ln x)2x}{x^4}$ M1

$\dfrac{d^2y}{dx^2} = \dfrac{-x-2x+2x\ln x}{x^4} = \dfrac{2\ln x - 3}{x^3}$

At $\left(e, \dfrac{1}{e}\right)$, $\dfrac{d^2y}{dx^2} == \dfrac{2-3}{e^3} < 0$ M1

$\therefore \; \left(e, \dfrac{1}{e}\right)$ is a maximum point A1

(b) At point of inflexion, $\dfrac{d^2y}{dx^2} = \dfrac{2\ln x - 3}{x^3} = 0$

$2\ln x = 3$

$\ln x = \dfrac{3}{2}$

$x = e^{\frac{3}{2}}$

$y = \dfrac{1}{e^{\frac{3}{2}}}\ln\dfrac{3}{2} = \dfrac{3}{2e^{\frac{3}{2}}}$

8. The functions f and g are defined as follows

$$f : x \to \ln(3x+2)$$

$$g : x \to \frac{e^x - 2}{3}$$

(a) Show algebraically that f is a one-to-one function. [3 marks]

(b) Find the composite function $f \circ g$ and $g \circ f$.

Hence, state the relationship between the functions f and g [5 marks]

(c) On the same axes, sketch the graphs of f and g and state the domain and range of f and g. [7 marks]

Scheme

(a) Let $f(x_1) = f(x_2)$

$\ln(3x_1 + 2) = \ln(3x_2 + 2)$ M1

$3x_1 + 2 = 3x_2 + 2$

$x_1 = x_2$ A1

Since $x_1 = x_2$, f is a one-to-one function A1

(b) $f \circ g = fg(x)$

$= f\left(\frac{e^x - 2}{3}\right)$

$= \ln\left(3\left(\frac{e^x - 2}{3}\right) + 2\right)$ M1

$= \ln(e^x - 2 + 2) = \ln e^x = x$ A1

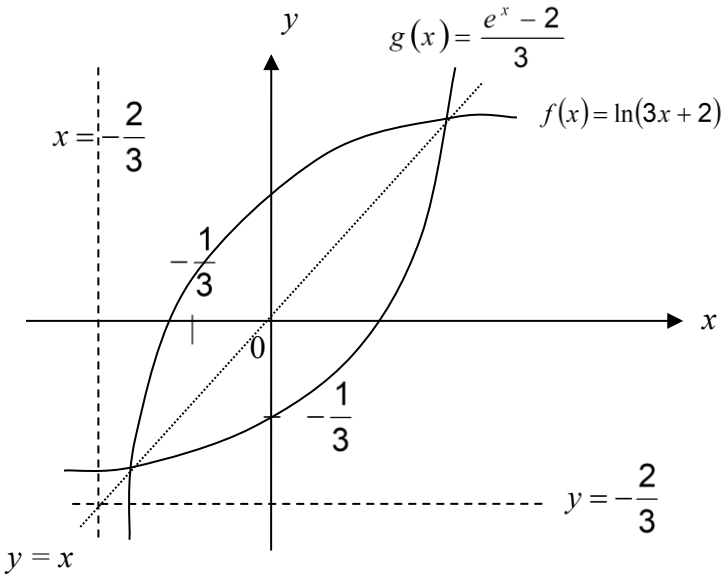
$g \circ f = gf(x)$

$= g(\ln 3x + 2)$

$= \frac{e^{\ln(3x+2)} - 2}{3}$ M1

$= \frac{3x + 2 - 2}{2} = x$ A1

Since $f \circ g = g \circ f = x$, f is the inverse of g or vice-versa B1



D1 – shape for $f(x)$ or $g(x)$
D1 – asymptote of $f(x)$ or $g(x)$
D1 – all correct for $f(x)$ or $g(x)$
M1 – reflection of curve f(x) or g(x) at the line y = x
A1

$D_f = \left(-\frac{2}{3}, \infty\right)$ B1

$Rg = (-\infty, \infty)$ B1

TAMAT SAMPEL

Suka sampel ini?

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