SAMPEL PERCUMA

Trial STPM 2015 Mathematics T Semester 1 Kuala Lumpur

Set penuh: Kertas Percubaan STPM

Sem 1 · Mathematics T

Sampel ini: 7 muka surat. Set penuh ada:

- 3 fail PDF
- Semester 1: kertas percubaan (trial) negeri, subjek yang dipilih + skema
- Blueprint 30 hari Semester 1 (setiap subjek)
- Semester 3: percubaan 2024, 10 subjek, banyak negeri, soalan + skema dalam satu fail

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KL SMK METHODIST MathsT P1 2015 QA

Section A [45 *marks*]

Answer **all** *questions in this section.*

1 The functions *f* and g are defined as follows:

$$f(x) = x^2 - 3$$
$$g(x) = \sqrt{x+3}$$

(a) State the domains and ranges of *f* and *g*. [4 *marks*]

(b) Sketch the graphs of *f* and *g* on the same coordinate axes. [3 *marks*]

(c) State the domain of *f* that needs to be redefined so that *f* is the inverse of *g*. [1 *marks*]

2 Express $8\cos^2\theta - 6\sin\theta\cos\theta - 4$ as $R\cos(2\theta + \alpha)$. [3 *marks*]

Hence, solve the equation $8\cos^2\theta - 6\sin\theta\cos\theta = 7$ for $0° \le \theta \le 360°$. [3 *marks*]

3 Show that $\frac{1}{5r-2} - \frac{1}{5r+3} = \frac{A}{(5r-2)(5r+3)}$, stating the value of the constant A. [2 *marks*]

Hence use the method of difference to show that

$$\sum_{r=1}^{n} \frac{1}{(5r-2)(5r+3)} = \frac{n}{3(5n+3)}$$ [4 *marks*]

Find the value of $\sum_{r=1}^{\infty} \frac{1}{(5r-2)(5r+3)}$ [1 *marks*]

4 If $z = \cos\theta + i\sin\theta$, show that $\left(z + \frac{1}{z}\right)^5 = 32\cos^5\theta$. [3 *marks*]

Hence, prove that $\cos^5\theta = \frac{1}{16}(\cos 5\theta + 5\cos 3\theta + 10\cos\theta)$. [5 *marks*]

5 Given the matrix $\mathbf{A} = \begin{pmatrix} 1 & 4 & -2 \\ -1 & 1 & -1 \\ p & 0 & 1 \end{pmatrix}$, where $p \neq \frac{5}{2}$

(a) Show that **A** is non-singular. [3 *marks*]

(b) By using elementary row operations, find $\mathbf{A}^{-1}$ given $p = 3$. [4 *marks*]

(c) Hence, solve the following system of linear equations:

$$x + 4y - 2z = 5$$
$$y - z = x$$
$$z = 8 - 3x$$

[2 *marks*]

6 The sum of distance of a moving point P from the point (0, 4) and the distance from the origin is 8 units. Find the equation of the locus of P and show that it is an ellipse. Sketch the graph of the locus. [7 *marks*]

Section B [15 *marks*]

Answer any **one** *questions in this section.*

7 Expand $(1-x)^{-p}\left(1-x^2\right)^{-q}$ in ascending powers of x up to and including the term in x^3.[6 *marks*]

(a) If the first three terms in the expansion above are $1-2x+\frac{10}{9}x^2$, find the values of p and q.

[3 *marks*]

(b) If $q=-\frac{1}{8}$ and the value of p is such that the coefficient of x^2 has the smallest value, show that the coefficient of x^3 in the above expansion is zero. [6 *marks*]

8 The points A, B and C have position vectors $\underline{a}$, $\underline{b}$ and $\underline{c}$ respectively, relative to an origin O, where

$$\underline{a}=3\underline{i}+4\underline{j}+5\underline{k},\ \underline{b}=4\underline{i}+6\underline{j}+7\underline{k},\ \underline{c}=\underline{i}+5\underline{j}+3\underline{k}$$

(a) Find $(\underline{b}-\underline{a})\times(\underline{c}-\underline{a})$. [3 *marks*]

(b) Hence, or otherwise, find the area of ΔABC and the volume of tetrahedron $OABC$.

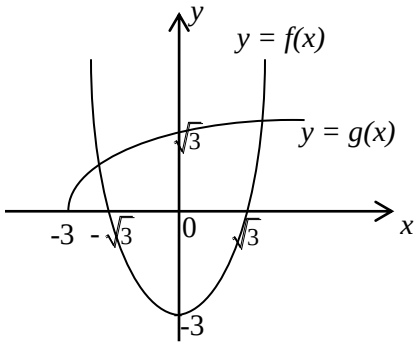
[4 *marks*]

(c) Find an equation of the plane ABC in the form $\underline{r}\bullet\underline{n}=p$. [2 *marks*]

Given that the point D has position vector $(2\underline{i}+\underline{j}+2\underline{k})$,

(d) find the coordinates of the point of intersection, E, of $\overrightarrow{OD}$ with the plane ABC. [3 *marks*]

(e) find the acute angle between $\overrightarrow{ED}$ and the plane ABC. [3 *marks*]

No	Working	Marks
1	**(a)** $\mathbf{D_f = \{x : x \in R\}}$ $\mathbf{R_f = \{y : y \geq -3, y \in R\}}$ $\mathbf{D_g = \{x : x \geq -3, x \in R\}}$ $\mathbf{R_g = \{y : y \geq 0, y \in R\}}$	B1 B1 B1 B1
	(b) 	D1 ($y=f$) D1 ($y=g$) D1 (*all correct*)
	(c) $D_f = \mathbf{\{x : x \geq 0, x \in R\}}$	B1
	Total	***8 marks***
2	$8\cos^2\theta - 6\sin\theta\cos\theta - 4$ $= 4\left(2\cos^2\theta - 1\right) - 3\left(\sin 2\theta\right)$ $= 4\cos 2\theta - 3\sin 2\theta$ $= 5\cos\left(2\theta + 36.9°\right)$ *accept 36°52'*	M1 M1 A1
	$8\cos^2\theta - 6\sin\theta\cos\theta = 7$ $8\cos^2\theta - 6\sin\theta\cos\theta - 4 = 3$ $5\cos\left(2\theta + 36.9°\right) = 3$ $\cos\left(2\theta + 36.9°\right) = 0.6$ $2\theta + 36.9° = 53.1°, 306.9°, 413.1°, 666.9°$ $\theta = \mathbf{8.1°, 135°, 188.1°, 315°}$	M1 M1 A1
	Total	***6 marks***

3	$\frac{1}{5r-2}-\frac{1}{5r+3}=\frac{5r+3-5r+2}{(5r-2)(5r+3)}=\frac{5}{(5r-2)(5r+3)}$ **A = 5** $\sum_{r=1}^{n}\frac{1}{(5r-2)(5r+3)}=\frac{1}{5}\sum_{r=1}^{n}\frac{5}{(5r-2)(5r+3)}$ $=\frac{1}{5}\sum_{r=1}^{n} f(r)-f(r+1)$ $=\frac{1}{5}[f(1)-f(n+1)]$ $=\frac{1}{5}\left[\frac{1}{3}-\frac{1}{5(n+1)-2}\right]$ $=\frac{1}{5}\left[\frac{1}{3}-\frac{1}{5n+3}\right]=\frac{\boldsymbol{n}}{\mathbf{3}(\mathbf{5}\boldsymbol{n}+\mathbf{3})}$ $\sum_{r=1}^{8}\frac{1}{(5r-2)(5r+3)}=\frac{\mathbf{1}}{\mathbf{15}}$ **Total**	M1 A1 M1 M1 M1 A1 B1 ***7 marks***
4	Let $z = \cos\theta + i\sin\theta$ $\frac{1}{z}=\cos\theta - i\sin\theta$ $z+\frac{1}{z}=2\cos\theta$ $\left(z+\frac{1}{z}\right)^5=(2\cos\theta)^5$ $=32\cos^5\theta$ $\left(z+\frac{1}{z}\right)^5=z^5+\binom{5}{1}z^4.\frac{1}{z}+\binom{5}{2}z^3.\frac{1}{z^2}+\binom{5}{3}z^2.\frac{1}{z^3}+\binom{5}{4}z.\frac{1}{z^4}+\frac{1}{z^5}$ $=\left(z^5+\frac{1}{z^5}\right)+5\left(z^3+\frac{1}{z^3}\right)+10\left(z+\frac{1}{z}\right)$ $32\cos^5\theta=2\cos5\theta+5(2\cos3\theta)+10(2\cos\theta)$ $\cos^5\theta=\frac{1}{32}[2\cos5\theta+5(2\cos3\theta)+10(2\cos\theta)]$ $=\frac{1}{16}(\cos5\theta+5\cos3\theta+10\cos\theta)$ **Total**	 B1 M1 A1 M1 A1 M1M1 A1 **8 *marks***

5	(a) $\lvert\mathbf{A}\rvert = p\begin{vmatrix}4 & -2\\ 1 & -1\end{vmatrix} + \begin{vmatrix}1 & 4\\ -1 & 1\end{vmatrix}$	M1
	$= -2p + 5$	
	Since $p \neq \frac{5}{2}$, $-2p + 5 \neq 0$. $\therefore$ **A** is non-singular	M1A1
	(b) $\left(\begin{array}{ccc\|ccc}1 & 4 & -2 & 1 & 0 & 0\\ -1 & 1 & -1 & 0 & 1 & 0\\ 3 & 0 & 1 & 0 & 0 & 1\end{array}\right) \xrightarrow[3R_1 - R_3 \to R_3]{3R_2 + R_3 \to R_2} \left(\begin{array}{ccc\|ccc}1 & 4 & -2 & 1 & 0 & 0\\ 0 & 3 & -2 & 0 & 3 & 1\\ 0 & 12 & -7 & 3 & 0 & -1\end{array}\right)$	M1
	$\xrightarrow{R_3 - 4R_2 \to R_3} \left(\begin{array}{ccc\|ccc}1 & 4 & -2 & 1 & 0 & 0\\ 0 & 3 & -2 & 0 & 3 & 1\\ 0 & 0 & 1 & 3 & -12 & -5\end{array}\right)$	M1
	$\xrightarrow{R_1 - R_2 \to R_1} \left(\begin{array}{ccc\|ccc}1 & 1 & 0 & 1 & -3 & -1\\ 0 & 3 & -2 & 0 & 3 & 1\\ 0 & 0 & 1 & 3 & -12 & -5\end{array}\right)$	M1
	$\xrightarrow{\frac{1}{3}(R_2 + 2R_3) \to R_2} \left(\begin{array}{ccc\|ccc}1 & 1 & 0 & 1 & -3 & -1\\ 0 & 1 & 0 & 2 & -7 & -3\\ 0 & 0 & 1 & 3 & -12 & -5\end{array}\right)$	
	$\xrightarrow{R_1 - R_2 \to R_1} \left(\begin{array}{ccc\|ccc}1 & 0 & 0 & -1 & 4 & 2\\ 0 & 1 & 0 & 2 & -7 & -3\\ 0 & 0 & 1 & 3 & -12 & -5\end{array}\right)$	
	$\therefore \quad \mathbf{A}^{-1} = \begin{pmatrix}-1 & 4 & 2\\ 2 & -7 & -3\\ 3 & -12 & -5\end{pmatrix}$	A1
	$\begin{pmatrix}1 & 4 & -2\\ -1 & 1 & -1\\ 3 & 0 & 1\end{pmatrix}\begin{pmatrix}x\\ y\\ z\end{pmatrix} = \begin{pmatrix}5\\ 0\\ 8\end{pmatrix}$	
	(c) $\begin{pmatrix}x\\ y\\ z\end{pmatrix} = \begin{pmatrix}-1 & 4 & 2\\ 2 & -7 & -3\\ 3 & -12 & -5\end{pmatrix}\begin{pmatrix}5\\ 0\\ 8\end{pmatrix}$	M1
	$= \begin{pmatrix}11\\ -14\\ -25\end{pmatrix}$	
	$\therefore$ ***x* = 11, *y* = -14, *z* = -25**	A1
	Total	***9 marks***

7

$$(1-x)^{-p} = 1+(-p)(-x)+\frac{(-p)(-p-1)}{2!}(-x)^2+\frac{(-p)(-p-1)(-p-2)}{3!}(-x)^3+\ldots$$ M1

$$= 1+px+\frac{p(p+1)}{2}x^2+\frac{p(p+1)(p+2)}{6}x^3+\ldots$$ A1

$$(1-x^2)^{-q} = 1+(-q)(-x^2)+\frac{(-q)(-q-1)}{2!}(-x^2)^2+\frac{(-q)(-q-1)(-q-2)}{3!}(-x^2)^3+\ldots$$ M1

$$= 1+qx^2+\ldots$$ A1

$$(1-x)^{-p}(1-x^2)^{-q} = \left[1+px+\frac{p(p+1)}{2}x^2+\frac{p(p+1)(p+2)}{6}x^3+\ldots\right]\left[1+qx^2+\ldots\right]$$

$$= 1+px+\frac{p(p+1)}{2}x^2+\frac{p(p+1)(p+2)}{6} + qx^2 + pqx^3+\ldots$$ M1

$$= \mathbf{1+px+\frac{1}{2}\left[p(p+1)+2q\right]x^2+\frac{1}{6}\left[p(p+1)(p+2)+6pq\right]x^3+\ldots}$$ A1

(a) $px = -2x$

$\boldsymbol{p = -2}$ B1

$$\frac{1}{2}\left[-2(-2+1)+2q\right] = \frac{10}{9}$$ M1

$$\boldsymbol{q = \frac{1}{9}}$$ A1

(b) $q = -\frac{1}{8}$ coefficient of x^2 $= \frac{1}{2}\left[p(p+1)+2\left(-\frac{1}{8}\right)\right]$ M1

$$= \frac{1}{2}\left(p^2+p-\frac{1}{4}\right)$$

$$= \frac{1}{2}\left(\left(p+\frac{1}{2}\right)^2-\frac{1}{2}\right)$$ M1A1

The smallest value of the coefficient of x^2 occurs when $\boldsymbol{p = -\frac{1}{2}}$ A1

Coefficient of x^3 $= \frac{1}{6}\left[-\frac{1}{2}\left(-\frac{1}{2}+1\right)(-\frac{1}{2}+2)+6\left(-\frac{1}{2}\right)\left(-\frac{1}{8}\right)\right]$ M1

$$= \frac{1}{6}\left(-\frac{3}{8}+\frac{3}{8}\right)$$

$= 0$ A1

Total **15 *marks***

8	(a) $\underline{b}-\underline{a}=\begin{pmatrix}4\\6\\7\end{pmatrix}-\begin{pmatrix}3\\4\\5\end{pmatrix}=\underline{i}+2\underline{j}+2\underline{k}$ OR $\underline{c}-\underline{a}=\begin{pmatrix}1\\5\\3\end{pmatrix}-\begin{pmatrix}3\\4\\5\end{pmatrix}=-2\underline{i}+\underline{j}-2\underline{k}$	M1
	$(\underline{b}-\underline{a})\times(\underline{c}-\underline{a})=\begin{vmatrix}\underline{i}&\underline{j}&\underline{k}\\1&2&2\\-2&1&-2\end{vmatrix}$	M1
	$=$ **$-6\underline{i}-2\underline{j}+5\underline{k}$**	A1
	(b) Area of $\Delta ABC=\frac{1}{2}\left\|-6\underline{i}-2\underline{j}+5\underline{k}\right\|$	M1
	$=\frac{\mathbf{1}}{\mathbf{2}}\sqrt{\mathbf{65}}$ unit^2	A1
	Volume of tetrahedron $=\frac{1}{6}\begin{pmatrix}3\\4\\5\end{pmatrix}\bullet\begin{pmatrix}-6\\-2\\5\end{pmatrix}=\frac{1}{6}\left\|-18-8+25\right\|$	M1
	$=\frac{\mathbf{1}}{\mathbf{6}}$**unit^3**	A1
	(c) $\begin{pmatrix}3\\4\\5\end{pmatrix}\begin{pmatrix}-6\\-2\\5\end{pmatrix}=-18-8+25=-1$	M1
	Equation of plane ABC is $\underline{r}\left(6\underline{i}+2\underline{j}-5\underline{k}\right)=1$	A1
	(d) Equation of line OD : $\begin{pmatrix}0\\0\\0\end{pmatrix}+\lambda\begin{pmatrix}2\\1\\2\end{pmatrix}=\begin{pmatrix}2\lambda\\\lambda\\2\lambda\end{pmatrix}$	M1
	At point of intersection E, $\begin{pmatrix}2\lambda\\\lambda\\2\lambda\end{pmatrix}\begin{pmatrix}6\\2\\-5\end{pmatrix}=1$	
	$12\lambda+2\lambda-10\lambda=1$	
	$\lambda=\frac{1}{4}$	M1
	Coordinates of E are $\left(\frac{\mathbf{1}}{\mathbf{2}},\ \frac{\mathbf{1}}{\mathbf{4}},\ \frac{\mathbf{1}}{\mathbf{2}}\right)$	A1
	(e) $\overrightarrow{ED}=\begin{pmatrix}2\\1\\2\end{pmatrix}-\begin{pmatrix}\frac{1}{2}\\\frac{1}{4}\\\frac{1}{2}\end{pmatrix}=\begin{pmatrix}\frac{3}{2}\\\frac{3}{4}\\\frac{3}{2}\end{pmatrix}$	B1
	$\sin\theta=\frac{\begin{pmatrix}\frac{3}{2}\\\frac{3}{4}\\\frac{3}{2}\end{pmatrix}\begin{pmatrix}6\\2\\-5\end{pmatrix}}{\frac{9}{4}\times\sqrt{65}}=0.1654$	M1
	$\theta=9.52°$ **Total**	A1 **15 *marks***

TAMAT SAMPEL

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