SAMPEL PERCUMA

Trial STPM 2017 Mathematics T Semester 1 Johor (Laksamana)

Set penuh: Kertas Percubaan STPM

Sem 1 · Mathematics T

Sampel ini: 5 muka surat. Set penuh ada:

- 3 fail PDF
- Semester 1: kertas percubaan (trial) negeri, subjek yang dipilih + skema
- Blueprint 30 hari Semester 1 (setiap subjek)
- Semester 3: percubaan 2024, 10 subjek, banyak negeri, soalan + skema dalam satu fail

RM12.90

BELI SET PENUH RM12.90 >

nightmew.com/p/stpm · FPX semua bank · download sejurus lepas bayar

Sampel bermula di muka surat sebelah

Section A [45 *marks*]
Answer **all** *questions in this section.*

1 The functions *f* and *g* are defined as $f: x \to 2-x^2, x \in R, x \le a$ and $g: x \to \ln(3-x), x \in R \ x < 3$.

(*a*) Given that the inverse function of *f* exists, find the largest value of *a*. For this value of *a*, determine f^{-1} and state it's domain and range. [5 *marks*]

(*b*) Show that $g \circ f$ exists. [1 *mark*]

2 Express $h(x)=\dfrac{2}{(1+x)(1+x^2)}$ in partial fractions. [4 *marks*]

Hence, expand *h*(*x*) in ascending powers of x up to and including the term in x^3. [5 *marks*]

3 Given that matrices $H = \begin{pmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ a & b & 1 \end{pmatrix}$ and $K = \begin{pmatrix} 1 & 0 & 0 \\ z & 1 & 0 \\ x & y & 1 \end{pmatrix}$ where *K* is the inverse of *H*. Find the value of *z*. Express *x* and *y* in terms of *a* and *b*. [5 *marks*]

4 The complex number *z* is given by $z=\dfrac{\left[2\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right)\right]^4}{\left[\frac{1}{2}\left(\cos\frac{\pi}{6}-i\sin\frac{\pi}{6}\right)\right]^5}$. Find the modulus and argument of the complex number *z*. Hence, find the value of z^3. [7 *marks*]

5 The parametric equations of a conic given as $x = \cos 2\theta$ and $y = 4 \sin \theta$.

(*a*) Obtain the standard form for the equation of the conic. Name the conic. [3 *marks*]

(*b*) State the coordinates of the vertex, *V* and focus, *F* of the conic. [2 *marks*]

(c) Hence, sketch the conic. [2 *marks*]

6 The position vectors of the points *A*, *B*, *C* and *P* are $2\boldsymbol{i} + \boldsymbol{j} + 3\boldsymbol{k}$, $\boldsymbol{i} + 3\boldsymbol{j} + \boldsymbol{k}$, $3\boldsymbol{i} - 6\boldsymbol{j} + 6\boldsymbol{k}$ and $(2 - 2t)\boldsymbol{i} + (1 + 4t)\boldsymbol{j} + (3 - 4t)\boldsymbol{k}$ respectively.

(*a*) Find the angle *OBC*. [4 *marks*]

(*b*) Find the value of *t* if *CP* is perpendicular to *CB*. [3 *marks*]

(*c*) Show that for all the values of *t*, the points *P*, *A* and *B* are collinear. [4 *marks*]

Section B [15 *marks*]

Answer **one** *question only in this section.*

7 (*a*) The sum of first 5 terms of an arithmetic progression , where the common difference is non-zero, is 105. It is further given that the first, third and fourth terms of the arithmetic progression are the first 3 terms of a geometric progression.

Find the common difference of the arithmetic progression. [4 *marks*]

Deduce that the geometric progression is convergent. [2 *marks*]

(*b*) The sum of first *n* terms of a series is $\frac{a^{2-n}(b^n - a^n)}{b-a}, b \neq a$. Find the *n*-th term of the series. Hence, show that the series is a geometric series. [9 *marks*]

8 (*a*) Find the solution set for the inequality $\frac{5x}{x-2} \geq 3$. Hence, solve the inequality $\frac{5|x|}{|x|-2} \geq 3$. [6 *marks*]

(*b*) The position vectors of points *E*, *F*, *G* and *H* are $-3\boldsymbol{i}$, $-\boldsymbol{j} + 2\boldsymbol{k}$, $-3\boldsymbol{i} + 2\boldsymbol{k}$ and $\boldsymbol{i} + \boldsymbol{k}$ respectively.

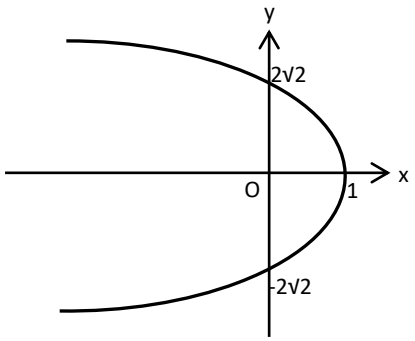
(i) Find the equation of plane *EFG* in the form $\boldsymbol{r} \cdot \boldsymbol{n} = d$. [4 *marks*]

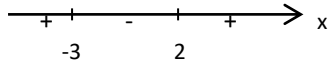
(ii) Find the perpendicular distance of *H* from the plane *EFG*. Hence, determine the volume of tetrahedron *EFGH*.

(You may assume that the volume of tetrahedron is $\frac{1}{3}$ x base area x height) [5 *marks*]

2017-1-JOH-LAKSAMANA-A

No.	Marking Scheme	Marks
1	The functions *f* and *g* are defined as *f*: $x \to 2-x^2, x \in R, x \le a$ and *g*: $x \to \ln(3-x), x \in R \ x < 3$. (*a*) Given that the inverse function of *f* exists, find the largest value of *a*. For this value of *a*, determine f^{-1} and state it's domain and range. [5 *marks*] (*b*) Show that $g \circ f$ exists. [1 *mark*]	
	(a) a = 0 Let f(x) = y $\therefore f^{-1}(x) = y$. $2 - x^2 = y$ $x^2 = 2 - y$ $x = \pm\sqrt{2-y} \Rightarrow x = -\sqrt{2-y}$ since $x \le 0$. $f^{-1}(y) = -\sqrt{2-y}$ $\therefore f^{-1}(x) = -\sqrt{2-x}$ $D_{f^{-1}} = \{x: x \le 2, x \in R\}$; $R_{f^{-1}} = \{y: y \le 0, y \in R\}$ (b) $R_f = (-\infty, 2] \subseteq D_g = (-\infty, 3)$ $\therefore g \circ f$ exists.	**1** **1** **1** **1, 1** **1**
2	Express $h(x) = \frac{2}{(1+x)(1+x^2)}$ in partial fractions. [4 *marks*] Hence, expand *h*(*x*) in ascending powers of x up to and including the term in x^3. [5 *marks*]	
	$\text{let } h(x) = \frac{A}{1+x} + \frac{Bx+C}{1+x^2}$ $A(1+x^2) + (Bx + C)(1+x) = 2$ When x = -1, A = 1 When x = 0, 1 + C = 2 $\therefore$ C =1 When x = 1, 2 + 2B + 2 = 2 $\therefore$ B = -1 $h(x) = \frac{1}{1+x} + \frac{1-x}{1+x^2}$ $h(x) = (1+x)^{-1} + (1-x)(1+x^2)^{-1}$ $(1+x)^{-1} = 1 + (-1)x + \frac{(-1)(-2)}{2!}x^2 + \frac{(-1)(-2)(-3)}{3!}x^3 + \ldots = 1 - x + x^2 - x^3 + \ldots$ $(1+x^2)^{-1} = 1 - x^2 + \ldots$ $h(x) = 1 - x + x^2 - x^3 + \ldots + (1-x)(1 - x^2 + \ldots)$ $= 1 - x + x^2 - x^3 + 1 - x^2 - x + x^3 + \ldots$ $= 2 - 2x + \ldots$	**1** **1** **1** (all 3 correct values) **1** (correct answer) **1** **1** **1** **1** **1** (correct answer)
3	Given that matrices $H = \begin{pmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ a & b & 1 \end{pmatrix}$ and $K = \begin{pmatrix} 1 & 0 & 0 \\ z & 1 & 0 \\ x & y & 1 \end{pmatrix}$ where *K* is the inverse of *H*. Find the value of *z*. Express *x* and *y* in terms of *a* and *b*. [5 *marks*]	
	HK = I $\begin{pmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ a & b & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ z & 1 & 0 \\ x & y & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ $\begin{pmatrix} 1 & 0 & 0 \\ 4+z & 1 & 0 \\ a+bz+x & b+y & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ 4+z = 0 $\therefore$ z = -4 a + bz + x = 0 $\therefore$ x = 4b - a b + y = 0 $\therefore$ y = -b	**1** **1** **1** **1** **1**
4	The complex number *z* is given by $z = \frac{\left[2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)\right]^4}{\left[\frac{1}{2}\left(\cos\frac{\pi}{6} - i\sin\frac{\pi}{6}\right)\right]^5}$. Find the modulus and argument of the complex number *z*. Hence, find the value of z^3. [7 *marks*]	
No.	**Marking Scheme**	**Marks**

	$\|z\| = \frac{2^4}{(0.5)^5} = 512$ units	**1, 1**
	arg(z) =4(π/3) -5(-π/6) = $\frac{13}{6}\pi$	**1, 1**
	= π/6 rad. (since -π ≤ arg(z) ≤ π)	**1**
	$z^3 = \left[512\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)\right]^3 = (512)^3\left(\cos\frac{3\pi}{6} + i\sin\frac{3\pi}{6}\right)$	**1**
	= 134217728*i*	**1**
5	The parametric equations of a conic given as $x = \cos 2\theta$ and $y = 4\sin\theta$. (*a*) Obtain the standard form for the equation of the conic. Name the conic. [3 *marks*] (*b*) State the coordinates of the vertex, *V* and focus, *F* of the conic. [2 *marks*] (c) Hence, sketch the conic. [2 *marks*]	
	(a) sin θ = y/4 $x = 1 - 2\sin^2\theta \ \therefore x = 1 - 2(y/4)^2$ $y^2 = 8(1 - x)$ $y^2 = 4(-2)(x - 1)$ Conic is parabola (b) V = (1, 0), F = (-1, 0) (c)  	**1** (substitution) **1** **1** **1, 1** **1, 1** (shape, points)
6	The position vectors of the points *A*, *B*, *C* and *P* are $2\boldsymbol{i} + \boldsymbol{j} + 3\boldsymbol{k}$, $\boldsymbol{i} + 3\boldsymbol{j} + \boldsymbol{k}$, $3\boldsymbol{i} - 6\boldsymbol{j} + 6\boldsymbol{k}$ and $(2 - 2t)\boldsymbol{i} + (1 + 4t)\boldsymbol{j} + (3 - 4t)\boldsymbol{k}$ respectively. (*a*) Find the angle *OBC*. [4 *marks*] (*b*) Find the value of *t* if *CP* is perpendicular to *CB*. [3 *marks*] (*c*) Show that for all the values of *t*, the points *P*, *A* and *B* are collinear. [4 *marks*]	
	(a) $\overrightarrow{OB}.\overrightarrow{CB} = (\boldsymbol{i} + 3\boldsymbol{j} + \boldsymbol{k}) \cdot (-2\boldsymbol{i} + 9\boldsymbol{j} - 5\boldsymbol{k}) = -2 + 27 - 5 = 20$ $\left\|\overrightarrow{OB}\right\| = \sqrt{1^2 + 3^2 + 1^2} = \sqrt{11}$ and $\left\|\overrightarrow{CB}\right\| = \sqrt{(-2)^2 + 9^2 + (-5)^2} = \sqrt{110}$ $\angle OBC = \cos^{-1}\frac{20}{\sqrt{11}x\sqrt{110}} = 54.9^0$ (b) $\overrightarrow{CP} = (-1 - 2t)\boldsymbol{i} + (7 + 4t)\boldsymbol{j} + (-3 - 4t)\boldsymbol{k}$; $\overrightarrow{CB} = -2\boldsymbol{i} + 9\boldsymbol{j} - 5\boldsymbol{k}$ $\overrightarrow{CP}.\overrightarrow{CB} = [(-1 - 2t)\boldsymbol{i} + (7 + 4t)\boldsymbol{j} + (-3 - 4t)\boldsymbol{k}] \cdot (-2\boldsymbol{i} + 9\boldsymbol{j} - 5\boldsymbol{k}) = 80 + 60t$ $80 + 60t = 0 \ \therefore t = -4/3$ (c) $\overrightarrow{BA} = \boldsymbol{i} - 2\boldsymbol{j} + 2\boldsymbol{k}$; $\overrightarrow{PA} = 2t\boldsymbol{i} - 4t\boldsymbol{j} + 4t\boldsymbol{k}$ $\overrightarrow{PA} = 2t\ \overrightarrow{BA}$ A is the common point. ∴ For all the values of t, the points P, A and B are collinear.	**1** **1** (either one) **1** (substitution), **1** (answer) **1** (either one) **1** (scalar product) **1** (answer) **1, 1** **1** **1**
7	(*a*) The sum of first 5 terms of an arithmetic progression, where the common difference is non-zero, is 105. It is further given that the first, third and fourth terms of the arithmetic progression are the first 3 terms of a geometric progression. Find the common difference of the arithmetic progression. [4 *marks*] Deduce that the geometric progression is convergent. [2 *marks*] (*b*) The sum of first *n* terms of a series is $\frac{a^{2-n}(b^n - a^n)}{b - a}, b \neq a.$ Find the *n*-th term of the series. Hence, show that the series is a geometric series. [9 *marks*]	
No.	**Marking Scheme**	**Marks**

	(a) $\frac{5}{2}(2a+4d) = 105 \ \therefore a+2d=21$	**1** (a + 2d = 21)
	G.P. : a, a + 2d, a + 3d = 21 - 2d, 21, 21 + d	**1**
	$\frac{21}{21-2d}=\frac{21+d}{21}$	**1** (concept of r)
	$\therefore 21^2=(21-2d)(21+d)$	
	$2d^2+21d=0$	**1** (value of d)
	$d(2d+21)=0$	
	d = 0 (rejected) or $d=-\frac{21}{2}$	**1** (value of r)
	r for G.P. = $\frac{21+(-10.5)}{21}=\frac{1}{2}$	
	Since $\|r\| < 1$, G.P. is convergent.	**1** (conclusion)
	(b) $S_{n-1}=\frac{a^{2-(n-1)}(b^{n-1}-a^{n-1})}{b-a}=\frac{a^{3-n}(b^{n-1}-a^{n-1})}{b-a}$	**1** (S_{n-1})
	$U_n=\frac{a^{2-n}(b^n-a^n)}{b-a}-\frac{a^{3-n}(b^{n-1}-a^{n-1})}{b-a}$	**1**
	$=\frac{a^{2-n}b^n-a^2-a^{3-n}b^{n-1}+a^2}{b-a}=\frac{a^{2-n}b^n-a^{3-n}b^{n-1}}{b-a}$	**1** (expanding)
	$==\frac{a^{2-n}b^{n-1}(b-a)}{b-a} \ \therefore U_n=a^{2-n}b^{n-1}$	**1** (simplifying)
	$U_{n-1}=a^{2-(n-1)}b^{(n-1)-1}=a^{3-n}b^{n-2}$	**1**(factorizing), **1** (answer)
	$\frac{U_n}{U_{n-1}}=\frac{a^{2-n}b^{n-1}}{a^{3-n}b^{n-2}}$	**1**
	$\therefore \frac{U_n}{U_{n-1}}=a^{2-n-(3-n)}b^{n-1-(n-2)}=b/a$	**1** (b/a)
	$\frac{U_n}{U_{n-1}}$ is a constant free from n. $\therefore$ the series is a geometric series.	**1** (conclusion)
8	(*a*) Find the solution set for the inequality $\frac{5x}{x-2}\geq 3$. Hence, solve the inequality $\frac{5\|x\|}{\|x\|-2}\geq 3$. [6 *marks*] (*b*) The position vectors of points *E*, *F*, *G* and *H* are $-3\boldsymbol{i}$, $-\boldsymbol{j}+2\boldsymbol{k}$, $-3\boldsymbol{i}+2\boldsymbol{k}$ and $\boldsymbol{i}+\boldsymbol{k}$ respectively. (i) Find the equation of plane *EFG* in the form $\boldsymbol{r}\cdot\boldsymbol{n}=d$. [4 *marks*] (ii) Find the perpendicular distance of *H* from the plane *EFG*. Hence, determine the volume of tetrahedron *EFGH*. (You may assume that the volume of tetrahedron is $\frac{1}{3}$ x base area x height) [5 *marks*]	
	(a) $\frac{5x}{x-2}-\frac{3(x-2)}{x-2}\geq 0$	**1**
	$\frac{x+3}{x-2}\geq 0$	**1**
		**1**
	$\{x : x\leq -3 \text{ or } x>2\}$	**1** (both answers correct)
	$\|x\|\leq -3$ (rejected) or $\|x\| > 2$	**1**
	{ x : x < -2 or x > 2 }	**1** (both answers correct)
	(b) i. $\vec{EF}=\vec{OF}-\vec{OE}=-\boldsymbol{j}+2\boldsymbol{k}-(-3\boldsymbol{i})=3\boldsymbol{i}-\boldsymbol{j}+2\boldsymbol{k}$	
	$\vec{EG}=\vec{OG}-\vec{OE}=-3\boldsymbol{i}+2\boldsymbol{k}-(-3\boldsymbol{i})=2\boldsymbol{k}$	**1** (either one of the vector)
	$\vec{EF}\times\vec{EG}=-2\boldsymbol{i}-6\boldsymbol{j}$	**1**
	$\boldsymbol{r}\cdot(-2\boldsymbol{i}-6\boldsymbol{j})=(-3\boldsymbol{i})\cdot(-2\boldsymbol{i}-6\boldsymbol{j})$	**1**
	$\boldsymbol{r}\cdot(-2\boldsymbol{i}-6\boldsymbol{j})=6 \quad \therefore \boldsymbol{r}\cdot(\boldsymbol{i}+3\boldsymbol{j})=-3$	**1** (correct answer)
	ii. Cartesian equation of plane: x + 3y +3 =0	
No.	**Marking Scheme**	**Marks**

TAMAT SAMPEL

Suka sampel ini?

Set penuh: Kertas Percubaan STPM

Sem 1 · Mathematics T

Sampel ini: 5 muka surat. Set penuh ada:

- 3 fail PDF
- Semester 1: kertas percubaan (trial) negeri, subjek yang dipilih + skema
- Blueprint 30 hari Semester 1 (setiap subjek)
- Semester 3: percubaan 2024, 10 subjek, banyak negeri, soalan + skema dalam satu fail

RM12.90

BELI SET PENUH RM12.90 >

nightmew.com/p/stpm · FPX semua bank · download sejurus lepas bayar