SAMPEL PERCUMA

Trial STPM 2017 Mathematics T Semester 1 Sarawak (St Thomas)

Set penuh: Kertas Percubaan STPM

Sem 1 · Mathematics T

Sampel ini: 7 muka surat. Set penuh ada:

- 3 fail PDF
- Semester 1: kertas percubaan (trial) negeri, subjek yang dipilih + skema
- Blueprint 30 hari Semester 1 (setiap subjek)
- Semester 3: percubaan 2024, 10 subjek, banyak negeri, soalan + skema dalam satu fail

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Sampel bermula di muka surat sebelah

2017-1-SAR-ST THOMAS

Section A: [*45 marks*]

*Answer **all** questionsin this section.*

1.Given that $\log_2 x + 2\log_4 y = 3$, show that $xy = 8$. Hence, solve for x and y which satisfy the simultaneous equations $\log_{10}(2x + y) = 1$ and $\log_2 x + 2\log_4 y = 3$. [6 *marks*]

2. Expand $\left(1 + \frac{2x}{3}\right)^{\frac{3}{2}}$until the term in x^3 in ascending powers of x. State the range of x for which this expansion is valid. Hence, evaluate $\sqrt{1.004^3}$, correct to 5 decimal places. [6 *marks*]

3. Show that the matrix $\begin{pmatrix} 3 & -1 & 5 \\ 4 & 3 & 3 \\ 5 & -4 & -2 \end{pmatrix}$is non-singular. [2*marks*]

Hence

(a) find its inverse using elementary row operations. [4*marks*]

(b)solve the simultaneous

$3x - y + 5z = 4$

$4x + 3y + 3z = 3$

$5x - 4y - 2z = 3$ [3 *marks*]

4. Show that the equation $16x^2 + 4y^2 - 64x - 40y + 100 = 0$ represents an ellipse and findthecentre, foci and vertices. Hence, sketch its graph. [7*marks*]

5. A and B are two points which have position vectors $\overrightarrow{OA} = \mathbf{i} + 2\mathbf{j} - 2\mathbf{k}$, $\overrightarrow{OB} = 2\mathbf{i} - 3\mathbf{j} + 6\mathbf{k}$ respectively.The point P lies on AB such that AP : AB = λ : 1 where λ is a scalar.

Show that $\overrightarrow{OP} = (1 + \lambda)\mathbf{i} + (2 - 5\lambda)\mathbf{j} + (-2 + 8\lambda)\mathbf{k}$. [2*marks*]

(a)Given that $\overrightarrow{OP}$ is perpendicular to $\overrightarrow{AB}$, show that $\lambda = \frac{5}{18}$. [3 *marks*]

(b) Find the position vector of P for which the angle AOP and the angle POB are the same. [5 *marks*]

6. Given that$z = \dfrac{\left(\cos\frac{\pi}{12} + i\sin\frac{\pi}{12}\right)^4}{\left(\cos\frac{\pi}{10} - i\sin\frac{\pi}{10}\right)^5\left(\cos\frac{\pi}{18} + i\sin\frac{\pi}{18}\right)^3}$.

(a) Write z in polar form. [3 *marks*]

(b) Show that z is a cube root of one. [2*marks*]

(c) Find $(z^2 - 1)(z + 2)$ in the form $x + iy$ where x and y are exact real numbers. [2*marks*]

Section B : [15 *marks***]**

*Answer **one** question only in this section.*

7. (a) (i) Express $\frac{r+4}{r(r+1)(r+2)}$ in partial fractions. [4*marks*]

(ii) Show that $\sum_{r=1}^{n} \frac{r+4}{r(r+1)(r+2)} = \frac{3}{2} - \frac{2}{n+1} + \frac{1}{n+2}$ [4*marks*]

(iii) Find the value of $\frac{5}{1\text{x }2\text{ x }3} + \frac{6}{2\text{ x }3\text{ x }4} + \frac{7}{3\text{ x }4\text{ x }5} + \cdots + \frac{12}{8\text{ x }9\text{ x }10}$. [3*marks*]

(b) Find the term independent of x in the expansion of $\left(x^2 + \frac{2}{x}\right)^6$. [4*marks*]

8. Given that the complex number $\alpha = 1 + 4i$ satisfies the cubic equation $z^3 + 5z^2 + kz + m = 0$, where k and m are real constants.

a) Find α^2 and α^3 in the form $a + bi$. [2 *marks*]

b) Find the value of k and show that $m = 119$. [4 *marks*]

c) Find the other two roots of the cubic equation. [4 *marks*]

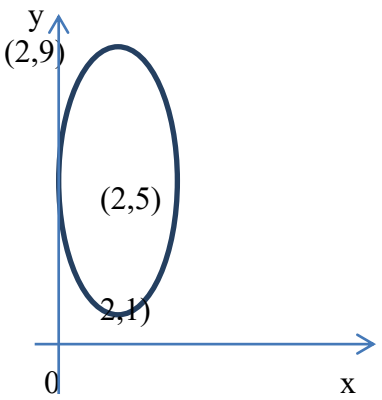
d) Illustrate the three roots of the cubic equation on an Argand diagram and find the argument of each root. [3 *marks*]

e) Verify that there is a constant c such that all three roots of the cubic equation satisfy $|z + 2| = c$. [2 *marks*]

2017 -1-SAR-ST Thomas

NO	SCHEME	MARKS
1	$\log_2 x + 2\log_4 y = 3$ $\log_2 x + 2\frac{\log_2 y}{\log_2 4} = 3$ $\log_2 xy = 3$ $xy = 2^3 = 8$ (shown)	 M1 M1
	$\log_{10}(2x + y) = 1$ $2x + y = 10$ (1) $xy = 8$ (2) $2x + \frac{8}{x} = 10$ $2x^2 - 10x + 8 = 0$ $(x - 1)(x - 4) = 0$ $x = 1$ or 4	B1 M1 A1
	When x = 1 , y = 8 When x = 4, y= 2	
	x =1, y = 8; x=4, y =2	A1
2	$\left(1 + \frac{2x}{3}\right)^{\frac{3}{2}} = 1 + \frac{3}{2}\left(\frac{2x}{3}\right) + \frac{\frac{3}{2}\left(\frac{1}{2}\right)}{2!}\left(\frac{2x}{3}\right)^2 + \frac{\frac{3}{2}\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{3!}\left(\frac{2x}{3}\right)^3 + \cdots$ (1)	M1
	$= 1 + x + \frac{1}{6}x^2 - \frac{1}{54}x^3 + \cdots$	A1
	The expansion is valid provided that $\left\|\frac{2x}{3}\right\| < 1$ $-\frac{3}{2} < x < \frac{3}{2}$	 A1
	$\sqrt{1.004^3} = (1 + 0.004)^3$	
	$\frac{2x}{3} = 0.004$ $x = 0.006$	 B1
	Substitute x = 0.006 into (1) $\sqrt{1.004^3} = 1 + 0.006 + \frac{1}{6}(0.006)^2 - \frac{1}{54}(0.006)^3 + \cdots$	M1
	≈ 1.006006 $= 1.00601$ (5.d.p)	 A1
3.	Let **A** $= \begin{pmatrix} 3 & -1 & 5 \\ 4 & 3 & 3 \\ 5 & -4 & -2 \end{pmatrix}$ $\det\mathbf{A} = 3\begin{vmatrix} 3 & 3 \\ -4 & 2 \end{vmatrix} - (-1)\begin{vmatrix} 4 & 3 \\ 5 & -2 \end{vmatrix} + 5\begin{vmatrix} 4 & 3 \\ 5 & -4 \end{vmatrix}$ $= 3(6) + 1(-23) + 5(-31)$ $= -160 \neq 0$	 M1 A1
	Since the determinant of **A** is not zero, the matrix is singular.	A1
	$\left(\begin{array}{ccc\|ccc} 3 & -1 & 5 & 1 & 0 & 0 \\ 4 & 3 & 3 & 0 & 1 & 0 \\ 5 & -4 & -2 & 0 & 0 & 1 \end{array}\right) - R_2 + R_1 \rightarrow R_1 \left(\begin{array}{ccc\|ccc} -1 & -4 & 2 & 1 & -1 & 0 \\ 4 & 3 & 3 & 0 & 1 & 0 \\ 5 & -4 & -2 & 0 & 0 & 1 \end{array}\right)$	
	$-R_1 \rightarrow R_1 \left(\begin{array}{ccc\|ccc} 1 & 4 & -2 & -1 & 1 & 0 \\ 4 & 3 & 3 & 0 & 1 & 0 \\ 5 & -4 & -2 & 0 & 0 & 1 \end{array}\right)$	

	$\xrightarrow[-5R_1 + R_3 \rightarrow R_3]{-4R_1 + R_2 \rightarrow R_2} \left(\begin{array}{ccc\|ccc} 1 & 4 & -2 & -1 & 1 & 0 \\ 0 & -13 & 11 & 4 & -3 & 0 \\ 0 & -24 & 8 & 5 & -5 & 1 \end{array}\right)$	M1 two 0's
	$\xrightarrow{-\frac{1}{13}R_2 \rightarrow R_2} \left(\begin{array}{ccc\|ccc} 1 & 4 & -2 & -1 & 1 & 0 \\ 0 & 1 & -\frac{11}{13} & -\frac{4}{13} & \frac{3}{13} & 0 \\ 0 & -24 & 8 & 5 & -5 & 1 \end{array}\right)$	
	$\xrightarrow[24R_2 + R_3 \rightarrow R_3]{-4R_2 + R_1 \rightarrow R_1} \left(\begin{array}{ccc\|ccc} 1 & 0 & \frac{18}{13} & \frac{3}{13} & \frac{1}{13} & 0 \\ 0 & 1 & -\frac{11}{13} & -\frac{4}{13} & \frac{3}{13} & 0 \\ 0 & 0 & -\frac{160}{13} & -\frac{31}{13} & \frac{7}{13} & 1 \end{array}\right)$	M1 triangular '0'
	$\xrightarrow{-\frac{13}{40}R_3 \rightarrow R_3} \left(\begin{array}{ccc\|ccc} 1 & 0 & \frac{18}{13} & \frac{3}{13} & \frac{1}{13} & 0 \\ 0 & 1 & -\frac{11}{13} & -\frac{4}{13} & \frac{3}{13} & 0 \\ 0 & 0 & 1 & \frac{31}{160} & -\frac{7}{160} & -\frac{13}{160} \end{array}\right)$	
	$\xrightarrow[-\frac{18}{13}R_3 + R_1 \rightarrow R_1]{\frac{11}{13}R_3 + R_2 \rightarrow R_2} \left(\begin{array}{ccc\|ccc} 1 & 0 & 0 & -\frac{6}{160} & \frac{22}{160} & \frac{18}{160} \\ 0 & 1 & 0 & -\frac{23}{160} & \frac{31}{160} & -\frac{11}{160} \\ 0 & 0 & 1 & \frac{31}{160} & -\frac{7}{160} & -\frac{13}{160} \end{array}\right)$	A1
	$\mathbf{A}^{-1} = \begin{pmatrix} -\frac{6}{160} & \frac{22}{160} & \frac{18}{160} \\ -\frac{23}{160} & \frac{31}{160} & -\frac{11}{160} \\ \frac{31}{160} & -\frac{7}{160} & -\frac{13}{160} \end{pmatrix}$	A1
	$\begin{pmatrix} 3 & -1 & 5 \\ 4 & 3 & 3 \\ 5 & -4 & -2 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 4 \\ 3 \\ 3 \end{pmatrix}$	
	$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 & -1 & 5 \\ 4 & 3 & 3 \\ 5 & -4 & -2 \end{pmatrix}^{-1} \begin{pmatrix} 4 \\ 3 \\ 3 \end{pmatrix}$	M1
	$= \frac{1}{160} \begin{pmatrix} -6 & 22 & 18 \\ -23 & 31 & -11 \\ 31 & -7 & -13 \end{pmatrix}^{-1} \begin{pmatrix} 4 \\ 3 \\ 3 \end{pmatrix}$	M1

	$= \frac{1}{160}\begin{pmatrix} 96 \\ -32 \\ 64 \end{pmatrix}$	
	$= \begin{pmatrix} \frac{3}{5} \\ -\frac{1}{5} \\ \frac{2}{5} \end{pmatrix}$	
	$x = \frac{3}{5}$, $y = -\frac{1}{5}$, $z = \frac{2}{5}$	A1
4	$16x^2 + 4y^2 - 64x - 40y + 100 = 0$ $16(x^2 - 4x) + 4(y^2 - 10y) = -100$ $16(x-2)^2 + 4(y-5)^2 = 64$ $\frac{(x-2)^2}{4} + \frac{(y-5)^2}{16} = 1$ Ellipse	M1 A1
	The centre of the ellipse is (2, 5)	**A1**
	$a^2 = 16$, $b^2 = 4$ $c^2 = a^2 - b^2 = 16 - 4 = 12$ $c = \sqrt{12} = 2\sqrt{3}$ The foci are $(2, 5 + 2\sqrt{3})$ and $(2, 5 + 2\sqrt{3})$ The vertices are (2, 9) and (2, 1)	 A1 A1
		A1 shape Intersect with y –axis A1 label
5	$\overrightarrow{0A} = \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix} \overrightarrow{0B} = \begin{pmatrix} 2 \\ -3 \\ 6 \end{pmatrix}$ A λ P $1-\lambda$ B AP: AB = λ : 1 AP : PB = λ : 1 - λ	
	$\overrightarrow{OP} = \frac{\lambda\overrightarrow{OB} + (1-\lambda)\overrightarrow{OA}}{\lambda + (1-\lambda)}$ $= \lambda\begin{pmatrix} 2 \\ -3 \\ 6 \end{pmatrix} + (1-\lambda)\begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix}$ $= \begin{pmatrix} 1+\lambda \\ 2-5\lambda \\ -2+8\lambda \end{pmatrix}$ $= (1+\lambda)\mathbf{i} + (2-5\lambda)\mathbf{j} + (-2+8\lambda)\mathbf{k}$	M1 A1

5a)	$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = \begin{pmatrix} 2 \\ -3 \\ 6 \end{pmatrix} - \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix} = \begin{pmatrix} 1 \\ -5 \\ 8 \end{pmatrix}$ $\overrightarrow{OP} \,.\overrightarrow{AB} = 0$ $\begin{pmatrix} 1+\lambda \\ 2-5\lambda \\ -2+8\lambda \end{pmatrix}.\begin{pmatrix} 1 \\ -5 \\ 8 \end{pmatrix} = 0$ $1(1+\lambda) - 5(2-5\lambda) + 8(-2+8\lambda) = 0$ $-25 + 90\lambda = 0$ $\lambda = \frac{5}{18}$	M1 M1 A1
(b)	Angle AOP = angle POB $\frac{\overrightarrow{OA}.\overrightarrow{OP}}{\lvert\overrightarrow{OA}\rvert\lvert\overrightarrow{OP}\rvert} = \frac{\overrightarrow{OB}.\overrightarrow{OP}}{\lvert\overrightarrow{OB}\rvert\lvert\overrightarrow{OP}\rvert}$ $\frac{\begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix}.\begin{pmatrix} 1+\lambda \\ 2-5\lambda \\ -2+8\lambda \end{pmatrix}}{\sqrt{1^2+2^2+(-2)^2}} = \frac{\begin{pmatrix} 2 \\ -3 \\ 6 \end{pmatrix}.\begin{pmatrix} 1+\lambda \\ 2-5\lambda \\ -2+8\lambda \end{pmatrix}}{\sqrt{2^2+(-3)^2+6^2}}$ $\frac{1+\lambda+2(2-5\lambda)-2(-2+8\lambda)}{\sqrt{9}} = \frac{2(1+\lambda)-3(2-5\lambda)+6(-2+8\lambda)}{\sqrt{49}}$ $370\,\lambda = 111$ $\lambda = \frac{3}{10}$ $\overrightarrow{OP} =$ $= \begin{pmatrix} 1+0.3 \\ 2-5(0.3) \\ -2+8(0.3) \end{pmatrix}$ $= \begin{pmatrix} \frac{13}{10} \\ \frac{1}{2} \\ \frac{2}{5} \end{pmatrix}$ $= \frac{13}{10}\mathbf{i} + \frac{1}{2}\mathbf{j} + \frac{2}{5}\mathbf{k}$	M1 M1 A1 M1 A1
6 (a)	$z = \frac{\left(\cos\frac{\pi}{12} + i\sin\frac{\pi}{12}\right)^4}{\left(\cos\frac{\pi}{10} - i\sin\frac{\pi}{10}\right)^5\left(\cos\frac{\pi}{18} + i\sin\frac{\pi}{18}\right)^3}$ $= \frac{\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)}{\left(\cos\left(-\frac{\pi}{2}\right) + i\sin\left(-\frac{\pi}{2}\right)\right)\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)}$	M1

ii	$\sum_{r=1}^{n} \frac{r+4}{r(r+1)(r+2)} = \sum_{r=1}^{n} \left(\frac{2}{r} - \frac{3}{r+1} + \frac{1}{r+2}\right)$	
	$= \sum_{r=1}^{n} \left(\frac{2}{r} - \frac{2}{r+1} - \frac{1}{r+1} + \frac{1}{r+2}\right)$	M1
	$= -2\sum_{r=1}^{n} \left(\frac{1}{r+1} - \frac{1}{r}\right) + \sum_{r=1}^{n} \left(\frac{1}{r+2} - \frac{1}{r+1}\right)$	A1
	$= -2\left(\frac{1}{n+1} - 1\right) + \left(\frac{1}{n+2} - \frac{1}{2}\right)$	
	$= \frac{3}{2} - \frac{2}{n+1} + \frac{1}{n+2}$	A1
Iii	$\frac{5}{1\text{x }2\text{ x }3} + \frac{6}{2\text{ x }3\text{ x }4} + \frac{7}{3\text{ x }4\text{ x }5} + \cdots + \frac{12}{8\text{ x }9\text{ x }10}$	B1
	$=\sum_{r=1}^{8} \frac{r+4}{r(r+1)(r+2)}$	M1
	$=\frac{3}{2} - \frac{2}{8+1} + \frac{1}{8+2}$	A1
	$=1\frac{17}{45}$	
b	$\left(x^2 + \frac{2}{x}\right)^6$.	B1
	$U_{r+1} = \binom{6}{r} x^{2(6-r)} \left(\frac{2}{x}\right)^r$	
	$= \binom{6}{r} x^{12-2r} 2^r x^{-r}$	A1
	$= \binom{6}{r} 2^r x^{12-3r} x^{-r}$	
	The term independent of x is when $12 - 3r = 0$ $r = 4$	M1
	When r = 4, the required term is $\binom{6}{4} 2^4 = 240$	A1
8. (a)	$\alpha = 1 + 4i$ $\alpha^2 = (1 + 4i)^2$ $= -15 + 8i$	A1
	$\alpha^3 = (1 + 4i)^2(1 + 4i)$ $= -47 - 52i$	A1

TAMAT SAMPEL

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