SAMPEL PERCUMA

Trial STPM 2018 Mathematics T Semester 1 Johor (Johor Jaya)

Set penuh: Kertas Percubaan STPM

Sem 1 · Mathematics T

Sampel ini: 7 muka surat. Set penuh ada:

- 3 fail PDF
- Semester 1: kertas percubaan (trial) negeri, subjek yang dipilih + skema
- Blueprint 30 hari Semester 1 (setiap subjek)
- Semester 3: percubaan 2024, 10 subjek, banyak negeri, soalan + skema dalam satu fail

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Sampel bermula di muka surat sebelah

Section A[45 marks]
*Answer **all**questions in this section.*

1. The functions f and g are defined by $f: x \rightarrow \frac{2}{3-2x}$ for $x \in \mathbb{R}$, $x \neq \frac{3}{2}$ and $g: x \rightarrow 4x + a$ for $x \in \mathbb{R}$, where a is a constant.
   (a) Find an expression for $f^{-1}(x)$. [3 *marks*]
   (b) Find the possible values of a given that the equation $f^{-1}(x) = g^{-1}(x)$ has two equal roots. [4 *marks*]

2. Use the binomial expansions of $\left(\sqrt{2}+1\right)^6$ and $\left(\sqrt{2}-1\right)^6$ to evaluate $\left(\sqrt{2}+1\right)^6+\left(\sqrt{2}-1\right)^6$.
   Hence, show that $197 \leq \left(\sqrt{2}+1\right)^6 \leq 198$. [7*marks*]

3. The matrices **A** and **B** are given by

$$\mathbf{A}=\begin{pmatrix}-1 & 2 & 1\\ -3 & 1 & 4\\ 0 & 1 & 2\end{pmatrix}, \qquad \mathbf{B}=\begin{pmatrix}-35 & 19 & 18\\ -27 & -13 & 45\\ -3 & 12 & 5\end{pmatrix}.$$

   Find the matrix $\mathbf{A}^2\mathbf{B}$ and deduce the inverse of **A**. [6*marks*]
   Hence, solve the system of linear equations

$$\begin{aligned} x - 2y - z &= -8,\\ 3x - y - 4z &= -15,\\ y + 2z &= 4. \end{aligned}$$

[4*marks*]

4. (*a*) Express $z = 1 + \sqrt{3}i$ in the form $r(\cos\theta + i\sin\theta)$, where *r* is the modulus and θ is the argument of the complex number.
   Hence, show that $\left(1+\sqrt{3}i\right)^5 = 16 - 16\sqrt{3}i$. [5*marks*]

   (*b*) Express z^6z^* in the form of $x + yi$, where z^* is the conjugate of *z* and $x, y \in R$. [2*marks*]

5. The parametric equations of a curve are $x = t - \frac{1}{t}$ and $y = t + \frac{1}{t}$. By taking $y^2 - x^2$ or otherwise, show that the curve is a hyperbola [3*marks*]
   Find
   (a) The coordinates of the vertices and foci [2*marks*]
   (b) The equations of the asymptotes [1*marks*]
   (c) Sketch the curve [2*marks*]

6. The vertices of a triangle PQR have coordinates $P(2,-3,-1)$, $Q(4,5,1)$, $R(-1,-2,6)$.
   (a) Find the vectors $\overrightarrow{PQ}$ and $\overrightarrow{PR}$. [2*marks*]
   (b) Hence, using a vector product, find a unit vector that is normal to both vectors $\overrightarrow{PQ}$ and $\overrightarrow{PR}$. [4*marks*]

Section B [15 *marks*]
*Answer any***one***question in this section.*

7. (a) Solve the equation
$$\log_4(2x+3)+\log_4(2x+15)=1+\log_4(14x+5)$$ [5 *marks*]

(b) (i) Given that $4\sin x+5\cos x=0$, find the value of $\tan x$. [2 *marks*]

(ii) Hence, solve the equation $(1-\tan x)(4\sin x+5\cos x)=0$ in the interval $0^o \le x \le 360^o$, giving your values of x to the nearest 0.1^o. [3 *marks*]

(c) Prove that
$$\frac{16+9\sin^2\theta}{5-3\cos\theta}=5+3\cos\theta$$

Find the least value of $\frac{16+9\sin^2\theta}{5-3\cos\theta}$.

State the exact value of θ, in radians in the interval $0 \le \theta < 2\pi$, at which this least value occurs.

[5 *marks*]

8. (a) Find the expansion $\sqrt[3]{\frac{1-2x}{(1+3x)^2}}$ in ascending powers of x until the term in x^3. State the set of values of x for which the expansion is valid. [8 *marks*]

(b) If the expansion of $(1+ax)^n$ in ascending powers of x until the term in x^2 is $1+\frac{1}{15}x-\frac{1}{225}x^2$, find the values of a and n. [4 *marks*]

Using these values of a and n, and taking $x=0.08$, find the value of $\sqrt[3]{127}$ correct to four decimal places. [3 *marks*]

MATHEMATICAL FORMULAE

Trigonometrical identities

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$
$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Sum of series

For an arithmetic series

$$S_n = \frac{1}{2} n(a + l) = \frac{1}{2} n[2a + (n-1)d]$$

For a geometric series

$$S_n = \frac{a(1 - r^n)}{1 - r}, r \neq 1$$

Binomial expansions

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n, n \in Z^+$$
$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!} x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{r!} x^r + \ldots, n \in Q, |x| < 1$$

Conics

Parabola with vertex (h,k) and focus $(a+h,k)$ and directrix $x = -a + h$

$$(y-k)^2 = 4a(x-h)$$

Ellipse with centre (h,k) and foci $(-c+h,k), (c+h,k)$ where $c^2 = a^2 - b^2$

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

Hyperbola with centre (h,k) and foci $(-c+h,k), (c+h,k)$ where $c^2 = a^2 + b^2$

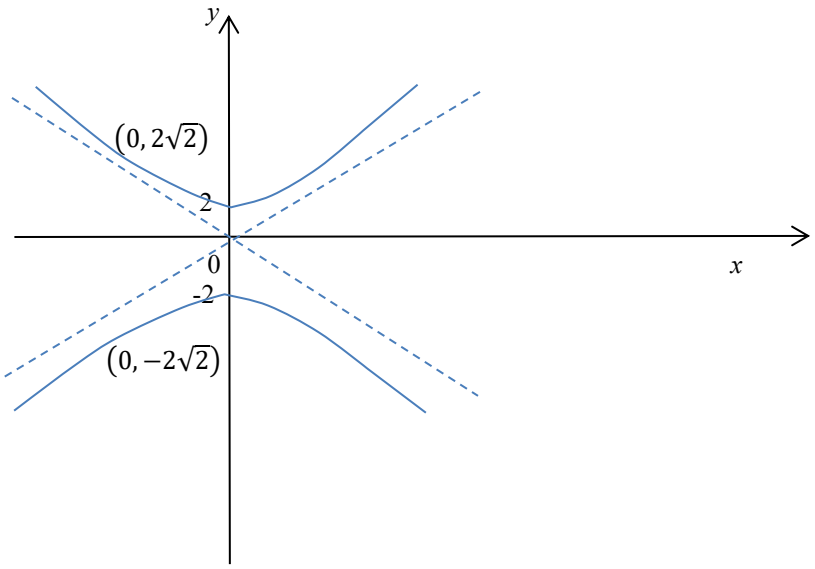
$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$

Marking Scheme PERCUBAAN STPM 2018 P 1 SEC (A)

1a)	1. (a) $f: x \rightarrow \frac{2}{3-2x}$, $x \in \mathbb{R}$, $x \neq \frac{3}{2}$ Let $f^{-1}(x) = y$ $f(y) = x$ $\frac{2}{3-2y} = x$ $3 - 2y = \frac{2}{x}$ $y = \frac{3}{2} - \frac{1}{x}$ $f^{-1}(x) = \frac{3}{2} - \frac{1}{x}$, $x \neq 0$ (b) $g: x \rightarrow 4x + a, x \in \mathbb{R}$ $g^{-1}(x) = \frac{x-a}{4}$, $x \in \mathbb{R}$ $f^{-1}(x) = g^{-1}(x)$ $\frac{3}{2} - \frac{1}{x} = \frac{x-a}{4}$ $\frac{3x-2}{2x} = \frac{x-a}{4}$ $x^2 - x(a+6) + 4 = 0$ Since $f^{-1}(x) = g^{-1}(x)$ has two equal roots, $(a+6)^2 - 16 = 0$ a = -2 or -10	M1 M1 A1 [3] B1 M1 M1 A1 [4]
2	$(\sqrt{2}+1)^6 = (\sqrt{2})^6 + \binom{6}{1}(\sqrt{2})^5(1) + \binom{6}{2}(\sqrt{2})^4(1)^2 + \binom{6}{3}(\sqrt{2})^3(1)^3 + \binom{6}{4}(\sqrt{2})^2(1)^4 + \binom{6}{5}(\sqrt{2})^1(1)^5 + (1)^6$ $(\sqrt{2}-1)^6 = (\sqrt{2})^6 - \binom{6}{1}(\sqrt{2})^5(1) + \binom{6}{2}(\sqrt{2})^4(1)^2 - \binom{6}{3}(\sqrt{2})^3(1)^3 + \binom{6}{4}(\sqrt{2})^2(1)^4 - \binom{6}{5}(\sqrt{2})^1(1)^5 + (1)^6$	B1 B1
	$(\sqrt{2}+1)^6 + (\sqrt{2}-1)^6$ $= 2\left[(\sqrt{2})^6 + \binom{6}{2}(\sqrt{2})^4(1)^2 + \binom{6}{4}(\sqrt{2})^2(1)^4 + (1)^6\right]$ $= 2(99) =$ 198	M1 A1
	$(\sqrt{2}+1)^6 = 198 - (\sqrt{2}-1)^6$	B1
	$1 < \sqrt{2} < 2$ $0 < \sqrt{2} - 1 < 1$	

	$0 < \left(\sqrt{2}-1\right)^6 < 1$ $-1 < -\left(\sqrt{2}-1\right)^6 < 0$ $197 < 198 - \left(\sqrt{2}-1\right)^6 < 198$ $197 < \left(\sqrt{2}+1\right)^6 < 198$	M1 A1
3	$A^2 = \begin{pmatrix}-1 & 2 & 1\\ -3 & 1 & 4\\ 0 & 1 & 2\end{pmatrix}\begin{pmatrix}-1 & 2 & 1\\ -3 & 1 & 4\\ 0 & 1 & 2\end{pmatrix} = \begin{pmatrix}-5 & 1 & 9\\ 0 & -1 & 9\\ -3 & 3 & 8\end{pmatrix}$ $A^2B = \begin{pmatrix}-5 & 1 & 9\\ 0 & -1 & 9\\ -3 & 3 & 8\end{pmatrix}\begin{pmatrix}-35 & 19 & 18\\ -27 & -13 & 45\\ -3 & 12 & 5\end{pmatrix} = \begin{pmatrix}121 & 0 & 0\\ 0 & 121 & 0\\ 0 & 0 & 121\end{pmatrix}$ $= 121\begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix}$ $A^2B = 121\ I$ $A^{-1}AAB = A^{-1}\ 121\ I$ $I\ AB = A^{-1}\ 121\ I$ $\mathbf{A^{-1}} = \frac{1}{121}AB$ $= \frac{1}{121}\begin{pmatrix}-1 & 2 & 1\\ -3 & 1 & 4\\ 0 & 1 & 2\end{pmatrix}\begin{pmatrix}-35 & 19 & 18\\ -27 & -13 & 45\\ -3 & 12 & 5\end{pmatrix}$ $= \frac{1}{121}\begin{pmatrix}-22 & -33 & 77\\ 66 & -22 & 11\\ -33 & 11 & 55\end{pmatrix}$ $= \begin{pmatrix}-\frac{2}{11} & -\frac{3}{11} & \frac{7}{11}\\ \frac{6}{11} & -\frac{2}{11} & \frac{1}{11}\\ -\frac{3}{11} & \frac{1}{11} & \frac{5}{11}\end{pmatrix}$	1 1 1 1 1 1 1 1 (Process of multiplication must be seen at least once, if not -1 mark)
	$\begin{pmatrix}-1 & 2 & 1\\ -3 & 1 & 4\\ 0 & 1 & 2\end{pmatrix}\begin{pmatrix}x\\ y\\ z\end{pmatrix} = \begin{pmatrix}8\\ 15\\ 4\end{pmatrix}$ $\begin{pmatrix}x\\ y\\ z\end{pmatrix} = \begin{pmatrix}-\frac{2}{11} & -\frac{3}{11} & \frac{7}{11}\\ \frac{6}{11} & -\frac{2}{11} & \frac{1}{11}\\ -\frac{3}{11} & \frac{1}{11} & \frac{5}{11}\end{pmatrix}\begin{pmatrix}8\\ 15\\ 4\end{pmatrix}$ $= \begin{pmatrix}-3\\ 2\\ 1\end{pmatrix}$ $x = -3$, $y = 2$ and $z = 1$	1 1 (allow premultiply with his A^{-1}) 1 1

4	$(x+yi)^2 = -5+12i$ $x^2 - y^2 + 2xyi = -5+12i$ $x^2 - y^2 = -5$.....(1) $2xy = 12$.....(2) From (2), $y = \frac{12}{2x}$ Substitute into (1), $x^2 - \left(\frac{12}{2x}\right)^2 = -5$ $x^4 + 5x^2 - 36 = 0$ $(x^2-4)(x^2+9) = 0$ $x^2 - 4 = 0$ $\quad$ $x^2 + 9 = 0$ $x^2 = 4$ $\quad$ (inadmissible) $x = \pm 2$ When $x = 2$, $\quad$ $x = -2$ $y = 3$ $\quad$ $y = -3$	1 (either one correct) 1 (attempt to solve) 1 1 (both pairs of values correct)
	$z^2 + 4z = -9+12i$ $(z+2)^2 - 4 = -9+12i$ $(z+2)^2 = -5+12i$ $(z+2)^2 = (2+3i)^2$ $\quad$ $(z+2)^2 = (-2-3i)^2$ $z = 3i$ $\quad$ $z = -4-3i$	1 (completing square LHS) 1 1 1 (both pairs of values correct)
5	$y^2 - x^2 = \left(t + \frac{1}{t}\right)^2 - \left(t - \frac{1}{t}\right)^2$ $= t^2 + 2 + \frac{1}{t^2} - t^2 + 2 - \frac{1}{t^2}$ $= 4$ $\frac{y^2}{4} - \frac{x^2}{4} = 1$ This is the standard form of a hyperbola	1 1 1
a) b)	$a = 2, b = 2, \quad c = \sqrt{a^2+b^2}$ $= \sqrt{2^2+2^2} = 2\sqrt{2}$ Vertices are (0, -2) and (0,2) Foci are $(0, -2\sqrt{2})$ *and* $(0, 2\sqrt{2})$ Asymptotes are $y = \pm\frac{a}{b}x$ $y = x$ or $y = -x$	1 1 1 (both equations)

c)		1 (Correct shape of hyperbola with vertical transverse axes) 1 (All correct – asymptotes passes through origin)
6a)	$\underset{PQ}{\rightarrow} = 2i + 8j + 2k$ $\underset{PR}{\rightarrow} = -3i + j + 7k$	1 1
b)	$\underset{PQ}{\rightarrow} \times \underset{PR}{\rightarrow} = \begin{vmatrix} i & j & k \\ 2 & 8 & 2 \\ -3 & 1 & 7 \end{vmatrix} = (56-2)i - (14+6)j + (2+24k)$ $= 54i - 20j + 26k$ Unit vector normal to $\underset{PQ}{\rightarrow}$ *and* $\underset{PR}{\rightarrow} = \frac{\overrightarrow{PQ} \times \overrightarrow{PR}}{\left\lvert \overrightarrow{PQ} \times \overrightarrow{PR} \right\rvert} = \frac{54i-20j+26k}{\sqrt{54^2+(-20)^2+26^2}}$ $= \frac{27}{\sqrt{998}}i - \frac{10}{\sqrt{998}}j + \frac{13}{\sqrt{998}}k$	1 1 1 (his $\frac{\overrightarrow{PQ} \times \overrightarrow{PR}}{\left\lvert \overrightarrow{PQ} \times \overrightarrow{PR} \right\rvert}$) 1

TAMAT SAMPEL

Suka sampel ini?

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Sem 1 · Mathematics T

Sampel ini: 7 muka surat. Set penuh ada:

- 3 fail PDF
- Semester 1: kertas percubaan (trial) negeri, subjek yang dipilih + skema
- Blueprint 30 hari Semester 1 (setiap subjek)
- Semester 3: percubaan 2024, 10 subjek, banyak negeri, soalan + skema dalam satu fail

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