SAMPEL PERCUMA

Trial STPM 2018 Mathematics T Semester 1 Negeri Sembilan (KGV)

Set penuh: Kertas Percubaan STPM

Sem 1 · Mathematics T

Sampel ini: 6 muka surat. Set penuh ada:

- 3 fail PDF
- Semester 1: kertas percubaan (trial) negeri, subjek yang dipilih + skema
- Blueprint 30 hari Semester 1 (setiap subjek)
- Semester 3: percubaan 2024, 10 subjek, banyak negeri, soalan + skema dalam satu fail

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Sampel bermula di muka surat sebelah

2018-1-NS-KGV

Section A [45 *marks*]
Answer **all** *questions.*

1. The functions f and g are defined by $f: x \rightarrow \frac{2}{3-2x}$ for $x \in \mathbb{R}$, $x \neq \frac{3}{2}$ and $g: x \rightarrow 4x + a$ for $x \in \mathbb{R}$, where a is a constant.
   (a) Find an expression for $f^{-1}(x)$. [3 *marks*]
   (b) Find the possible values of a given that the equation $f^{-1}(x) = g^{-1}(x)$ has two equal roots. [4 *marks*]

2. In the expansion of $(\frac{1}{ax} + 2ax^2)^5$, the coefficient of x is 5. Find the value of the constant a. [4*marks*]

3. Matrix **P** is given by $\mathbf{P} = \begin{pmatrix} 1 & 0 & 0 \\ 1 & -2 & 0 \\ -1 & 3 & 1 \end{pmatrix}$ for which $\mathbf{P}^2 + k\mathbf{P} + j\mathbf{I} = \mathbf{0}$.

   (a) Find the values of *k* and *j*. [5 *marks*]
   (b) Hence, deduce $\mathbf{P}^3$ and $\mathbf{P}^{-1}$. [5 *marks*]

4. Given that $z + 6iz = 5 - 7i$.
   (a) Find z in the polar form. [5 *marks*]
   (b) Use the de Moivre's theorem, show that $z^8 = 16$ [3 *marks*]

5. The line 2x + 5y -10 = 0 is a tangent to the circle at A(-2, 5). The centre of the circle lies in the line x + y – 4 = 0.
   (a) Find the centre and radius of the circle. Hence, write down the equation of the circle. [7*marks*]
   (b) Determine if the point P(3, -7) lies inside or outside the circle. [3 *marks*]

6. The points A and B have position vectors given by $\overrightarrow{OA}$ = **i** – 2**j** + 2**k** and $\overrightarrow{OB}$ = 3**i** + **j** + **k**. The line*l* has equation **r** = 2**i** + **j** + m**k** + μ(**i**– 2**j** – 4**k**), where m is a constant. Given that the line *l* intersects the line passing through A and B, find the value of m. [6*marks*]

Section B [15 *marks*]
*Answer***one** *question only.*

7 (a) The polynomial $p(x)$is defined by

$$p(x) = ax^3 + bx^2 - 17x - a,$$

where$a$ and b are constants. It is given that $(x+2)$ is a factor of $p(x)$and that the remainder is 28 when $p(x)$is divided by $(x-2)$.

(i) Find the values of $a$and b. [6*marks*]
(ii) Hence factorise $p(x)$completely. [2*marks*]
(iii) State the number of roots of the equation $p(2^y)=0$, justifying your answer. [2*marks*]

(b) Solve the equation 2ln(2x) – ln (x+3) =ln (3x+5). [5*marks*]

8. (a) Given that $u_r = \left(\frac{1}{4}\right)^{2r-3} + \left(\frac{1}{4}\right)^{2r-1}$.

(i) Express $\sum_{r=1}^{n} u_r$ in terms of $A\left(1 - \frac{B}{16^n}\right)$, where A and B are two constants to be determined. [7*marks*]

(ii) Determine the sum to infinity of this series. [2*marks*]

(b) Evaluate $\sum_{r=1}^{\infty} \frac{1}{10^{2r}}$. [3 *marks*]

(c) Hence, express 0.393939…(= $0.\overline{39}$)as a fraction in its lowest form. [3 *marks*]

2018-1-NS-KGV-MARKING SCHEMES

1. (a) $f: x \rightarrow \frac{2}{3-2x}$, $x \in \mathbb{R}$, $x \neq \frac{3}{2}$

Let $f^{-1}(x) = y$

$$f(y) = x$$

$\frac{2}{3-2y} = x$ M1

$$3 - 2y = \frac{2}{x}$$

$y = \frac{3}{2} - \frac{1}{x}$ M1

$f^{-1}(x) = \frac{3}{2} - \frac{1}{x}, x \neq 0$ A1 [3]

(b) $g: x \rightarrow 4x + a, x \in \mathbb{R}$

$g^{-1}(x) = \frac{x-a}{4}, x \in \mathbb{R}$ B1

$f^{-1}(x) = g^{-1}(x)$

$\frac{3}{2} - \frac{1}{x} = \frac{x-a}{4}$ M1

$$\frac{3x-2}{2x} = \frac{x-a}{4}$$

$$x^2 - x(a+6) + 4 = 0$$

Since $f^{-1}(x) = g^{-1}(x)$has two equal roots, $(a+6)^2 - 16 = 0$ M1

a = -2 or -10 A1 [4]

2. The term in $x = \binom{5}{2}(\frac{1}{ax})^3(2ax^2)^2$ B1

The coefficient of $x = 10(\frac{1}{a})^3(2a)^2 = 5$ M1M1

$a = 8$ A1 [4]

3. (a) $\mathbf{P}^2 = \begin{pmatrix} 1 & 0 & 0 \\ 1 & -2 & 0 \\ -1 & 3 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 1 & -2 & 0 \\ -1 & 3 & 1 \end{pmatrix}$ M1

$= \begin{pmatrix} 1 & 0 & 0 \\ -1 & 4 & 0 \\ 1 & -3 & 1 \end{pmatrix}$ A1

$\mathbf{P}^2 + k\mathbf{P} + j\mathbf{I} = \mathbf{0}$

$\begin{pmatrix} 1 & 0 & 0 \\ -1 & 4 & 0 \\ 1 & -3 & 1 \end{pmatrix} + k\begin{pmatrix} 1 & 0 & 0 \\ 1 & -2 & 0 \\ -1 & 3 & 1 \end{pmatrix} + j\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ M1

$1 - k = 0$ -----(1)

$k = 1$

$1 + k + j = 0$ -----(2) either equation M1

$1 + 1 + j = 0$

j = -2 both correct A1 [5]

(b) $\mathbf{P}^2 + k\mathbf{P} + j\mathbf{I} = \mathbf{0}$

$\mathbf{P}^2 + \mathbf{P} - 2\mathbf{I} = \mathbf{0}$

$\mathbf{P}^2 = 2\mathbf{I} - \mathbf{P}$

$\mathbf{P}^2\mathbf{P} = 2\mathbf{I}\mathbf{P} - \mathbf{P}\mathbf{P}$ M1

$\mathbf{P}^3 = 2\mathbf{P} - \mathbf{P}^2$

$= 2\mathbf{P} - (2\mathbf{I} - \mathbf{P})$

$= 3\mathbf{P} - 2\mathbf{I}$

$= 3\begin{pmatrix} 1 & 0 & 0 \\ 1 & -2 & 0 \\ -1 & 3 & 1 \end{pmatrix} - 2\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ M1

$= \begin{pmatrix} 1 & 0 & 0 \\ 3 & -8 & 0 \\ -3 & 9 & 1 \end{pmatrix}$ A1

$\mathbf{P}^2 = 2\mathbf{I} - \mathbf{P}$

$\mathbf{P}^2\mathbf{P}^{-1} = 2\mathbf{I}\mathbf{P}^{-1} - \mathbf{P}\mathbf{P}^{-1}$

$\mathbf{P} = 2\mathbf{P}^{-1} - \mathbf{I}$

$\mathbf{P}^{-1} = \frac{1}{2}(\mathbf{P} + \mathbf{I})$

$= \frac{1}{2}\left[\begin{pmatrix} 1 & 0 & 0 \\ 1 & -2 & 0 \\ -1 & 3 & 1 \end{pmatrix} + \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\right]$ M1

$= \begin{pmatrix} 1 & 0 & 0 \\ 0.5 & -0.5 & 0 \\ -0.5 & 1.5 & 1 \end{pmatrix}$ A1 [5]

4. (a) $z + 6iz = 5 - 7i$

$z(1 + 6i) = 5 - 7i$

$z = \frac{5-7i}{1+6i} \times \frac{1-6i}{1-6i}$ M1

$= \frac{-37-37i}{1+36}$

$= -1 - i$ A1

$|z| = \sqrt{(-1)^2 + (-1)^2}$

$= \sqrt{2}$ A1

$\text{Arg}z = -\pi + tan^{-1}1$

$= -\frac{3\pi}{4}$ A1

$z = \sqrt{2}\left[\cos\left(-\frac{3\pi}{4}\right) + isin\left(-\frac{3\pi}{4}\right)\right]$ A1

$= \sqrt{2}\left[\cos\frac{3\pi}{4} - isin\frac{3\pi}{4}\right]$ or A1 [5]

$z^8 = \left\{\sqrt{2}\left[\cos\left(-\frac{3\pi}{4}\right) + isin\left(-\frac{3\pi}{4}\right)\right]\right\}^8$ M1

$= 16\left[\cos 8\left(-\frac{3\pi}{4}\right) + isin\, 8\left(-\frac{3\pi}{4}\right)\right]$ M1

$= 16\,[\cos(-6\pi) + isin(-6\pi)]$

$= 16\,[1 + 0]$

$= 16$ A1 [3]

5. (a) Gradient of tangent = $-\frac{2}{5}$ B1

Let C(h,k) be the centre of circle.

$m_{AC} = \frac{k-5}{h+2} = \frac{5}{2}$

$2k - 10 = 5h + 10$

$2k - 5h = 20$ ----(1)

$h + k = 4$ ----- (2) either equation M1

$h = 4 - k$

$2k - 5(4 - k) = 20$ solve for h & k M1

$k = \frac{40}{7}$

$h = -\frac{12}{7}$

Therefore, centre of circle is $\left(-\frac{12}{7}, \frac{40}{7}\right)$ A1

Radius = $\sqrt{\left(-2 + \frac{12}{7}\right)^2 + \left(5 - \frac{40}{7}\right)^2}$ M1

$= \frac{\sqrt{29}}{7}$ A1

Equation of circle: $\left(x + \frac{12}{7}\right)^2 + \left(y - \frac{40}{7}\right)^2 = \frac{29}{49}$ A1 [7]

(b) Distance of P from the centre = $\sqrt{\left(3 + \frac{12}{7}\right)^2 + \left(-7 - \frac{40}{7}\right)^2}$ M1

$= \frac{\sqrt{9010}}{7}$

Since $\frac{\sqrt{9010}}{7} >$ radius$=\frac{\sqrt{29}}{7}$, therefore P lies outside the circle. M1A1 [3]

6. $\overrightarrow{AB} = \begin{pmatrix}3\\1\\1\end{pmatrix} - \begin{pmatrix}1\\-2\\2\end{pmatrix} = \begin{pmatrix}2\\3\\-1\end{pmatrix}$ M1

The line passing through A and B, **r** = $\begin{pmatrix}3\\1\\1\end{pmatrix} + \lambda\begin{pmatrix}2\\3\\-1\end{pmatrix}$ A1

$\begin{pmatrix}3\\1\\1\end{pmatrix} + \lambda\begin{pmatrix}2\\3\\-1\end{pmatrix} = \begin{pmatrix}2\\1\\m\end{pmatrix} + \mu\begin{pmatrix}1\\-2\\-4\end{pmatrix}$ M1

$3 + 2\lambda = 2 + \mu$

$\mu - 2\lambda = 1$----(1)

$1 + 3\lambda = 1 - 2\mu$ equate two pairs of component and solve M1

$\mu = -\frac{3}{2}\lambda$ (Substitute into (1))

$\lambda = -\frac{2}{7}$, $\mu = \frac{3}{7}$ A1

$1 + \frac{2}{7} = m - 4 \times \frac{3}{7}$

$m = 3$ A1 [6]

7a.(i)$p(-2) = a(-2)^3 + b(-2)^2 - 17(-2) - a = 0$ M1

$-9a + 4b = -34$-------(1)

$p(2) = a(2)^3 + b(2)^2 - 17(2) - a = 28$ M1

$7a + 4b = 62$-------(2) both (1) & (2) correct A1

$16a = 96$ solving equations M1

$a = 6$

$b = 5$ A1A1 [6]

(ii) $p(x) = 6x^3 + 5x^2 - 17x - 6$

$= (x + 2)(6x^2 - 7x - 3)$ M1

$= (x + 2)(3x + 1)(2x - 3)$ A1 [2]

(iii) $p(2^y) = 6(2^y)^3 + 5(2^y)^2 - 17(2^y) - 6 = 0$

$(2^y + 2)[3(2^y) + 1][2(2^y) - 3] = 0$

Since $2^y > 0 \; \forall \; y \in \mathbb{R}$ M1

therefore there is only 1 root. A1 [2]

b. $2\ln(2x) - \ln(x + 3) = \ln(3x + 5)$

$ln(2x)^2 - \ln(x + 3) = \ln(3x + 5)$ M1

$ln\frac{(2x)^2}{(x+3)} = \ln(3x + 5)$ M1

$\frac{(2x)^2}{(x+3)} = (3x + 5)$

$4x^2 = (3x + 5)(x + 3)$ M1

$= 3x^2 + 14x + 15$

$x^2 - 14x - 15 = 0$ M1

$(x - 15)(x + 1) = 0$

$x = 15 \text{or} x = -1$

Since $\ln(-2)$ is not defined, therefore $x = 15$ A1 [5]

8. (a) (i) $u_r = \left(\frac{1}{4}\right)^{2r-3} + \left(\frac{1}{4}\right)^{2r-1}$

$= \frac{4^3}{4^{2r}} + \frac{4}{4^{2r}}$ M1

$= 64\left(\frac{1}{4}\right)^{2r} + 4\left(\frac{1}{4}\right)^{2r}$

$= 68\left(\frac{1}{4}\right)^{2r}$ A1

$\sum_{r=1}^{n} u_r = 68\sum_{r=1}^{n}\left(\frac{1}{4}\right)^{2r}$

$= 68\left[\left(\frac{1}{4}\right)^2 + \left(\frac{1}{4}\right)^4 + \left(\frac{1}{4}\right)^6 + \cdots + \left(\frac{1}{4}\right)^{2n}\right]$ M1

$= 68\left(\frac{1}{4}\right)^2\left[1 + \left(\frac{1}{4}\right)^2 + \left(\frac{1}{4}\right)^4 + \cdots + \left(\frac{1}{4}\right)^{2(n-1)}\right]$

$= \frac{68}{16}\left[\frac{1-\left(\frac{1}{4}\right)^{2n}}{1-\left(\frac{1}{4}\right)^2}\right]$ M1

$= \frac{68}{15}\left[1 - \left(\frac{1}{4}\right)^{2n}\right]$

$= \frac{68}{15}\left(1 - \frac{1}{16^n}\right)$ A1

TAMAT SAMPEL

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- 3 fail PDF
- Semester 1: kertas percubaan (trial) negeri, subjek yang dipilih + skema
- Blueprint 30 hari Semester 1 (setiap subjek)
- Semester 3: percubaan 2024, 10 subjek, banyak negeri, soalan + skema dalam satu fail

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