SAMPEL PERCUMA

Trial STPM 2018 Mathematics T Semester 1 Pulau Pinang (Jit Sin)

Set penuh: Kertas Percubaan STPM

Sem 1 · Mathematics T

Sampel ini: 6 muka surat. Set penuh ada:

- 3 fail PDF
- Semester 1: kertas percubaan (trial) negeri, subjek yang dipilih + skema
- Blueprint 30 hari Semester 1 (setiap subjek)
- Semester 3: percubaan 2024, 10 subjek, banyak negeri, soalan + skema dalam satu fail

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Sampel bermula di muka surat sebelah

2018-1-PEN-JIT SIN

Section A [45 *marks*]

*Answer **all** questions in this section.*

1 Given that $3^{4-2x} = 5^{3y+x}$ and $3^{x-y} \cdot 25^{2y-1} = 1$, show that $x^2 - 3y^2 - 6xy + 4x + 16y - 8 = 0$.

[4 *marks*]

2 Express $\frac{2}{r^2 - 1}$ as partial fraction. [3 *marks*]

Hence, find a simple expression for $S = \sum_{r=2}^{n} \frac{2}{r^2 - 1}$ and determine whether S converges when *n* is large. [5 *marks*]

3 (a) The matrices **A, B** and **C** are given by

$$\mathbf{A} = \begin{pmatrix} 2 & 1 & 0 \\ -1 & 4 & 1 \\ 3 & 0 & -1 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} -3 & -1 & 2 \\ 1 & -1 & 0 \\ 1 & 1 & -1 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 7 & -2 & 6 \\ -4 & 6 & -1 \\ 12 & -1 & 3 \end{pmatrix}.$$

Find the matrix $(\mathbf{A} + \mathbf{B})\mathbf{C}$, and hence, deduce $(\mathbf{A} + \mathbf{B})^{-1}$. [5 *marks*]

(b) Using the result in (*a*), solve the system of linear equations

$$x - 2z = -17,$$
$$3y + z = 34,$$
$$4x + y - 2z = 17.$$

[5 *marks*]

4 The complex number *u* is defined by $u = \frac{5}{a + 2i}$, where the constant *a* is real.

(a) Express *u* in the form $x + iy$ where *x* and *y* are real. [2 *marks*]

(b) Find the value of *a* for which $\arg(u^*) = \frac{3\pi}{4}$, where * denotes the complex conjugate of *u*. [5 *marks*]

5 The equation of a conic is $9x^2 + 4y^2 + 54x + 32y + 109 = 0$.

(a) Obtain its standard form and identify the conic. [4 *marks*]

(b) Find the coordinates of its centre, vertices and foci. Hence, sketch its graph. [5 *marks*]

6 It is given that $\mathbf{a} = p\mathbf{i} - \mathbf{j} + 4\mathbf{k}$, $\mathbf{b} = q\mathbf{j} + 3\mathbf{k}$ and $\mathbf{a} \times \mathbf{b} = \mathbf{i} - 6\mathbf{j} - 2\mathbf{k}$, where p and q are constants.

(a) Determines the values of p and q. [3 *marks*]

(b) Find (i) the area of the parallelogram with sides **a** and **b**. [2 *marks*]

(ii) the angle between **a** and **b**. [2 *marks*]

Section B [15 *marks*]

Answer only **one** *question in this section.*

7 A polynomial $p(x) = 4x^4 + ax^3 + bx^2 + 8x - 3$ has a factor $x + 3$ and gives a remainder -10 when it is divided by $2x + 1$. Determine the values of the constants a and b. [5 *marks*]

Show that $\frac{1}{2}$ is the zero of $p(x)$ and factorise completely $p(x)$. [5 *marks*]

Hence, find the set of values of x for $p(x) < (2x - 1)(x + 3)(x - 2)$. [5 *marks*]

8 (a) The complex numbers z_1 and z_2 are given by $z_1 = 1 - 2i$ and $z_2 = 3 - i$. Find the modulus and argument of $\frac{z_2}{z_1}$. [5 *marks*]

Hence, determine the real value of $\left(\frac{z_2}{z_1}\right)^{20}$. [4 *marks*]

(b) The variable complex number z is given by $z = 2\cos\theta + i(1 - 2\sin\theta)$, where θ takes all the values in the interval $-\pi < \theta \leq \pi$.

(i) Show that $|z - i| = 2$ for all values of θ. [3 *marks*]

(ii) Prove that the real part of $\frac{1}{z + 2 - i}$ is a constant for $-\pi < \theta \leq \pi$. [3 *marks*]

2018-1-PEN-JIT SIN-MARKING SCHEME

Bil	Scheme	Marks	Tot
1	$3=5^{\frac{3y+x}{4-2x}}$ or $(4-2\text{x})\log 3=(3y+x)\log 5$	B1	4
	$3=5^{\frac{-4y+2}{x-y}}$ or $(\text{x}-y)\log 3=(-4y+2)\log 5$	B1	
	$\frac{3y+x}{4-2x}=\frac{-4y+2}{x-y}$ or $\frac{4-2\text{x}}{\text{x}-\text{y}}=\frac{3y+x}{-4y+2}$	M1	
	$x^2-3y^2-6xy+4x+16y-8=0$	A1	

2	$\frac{2}{r^2-1}\equiv\frac{A}{r-1}+\frac{B}{r+1}$	B1	8
	Try to solve $2=A(r+1)+B(r-1)$	M1	
	$\frac{2}{r^2-1}\equiv\frac{1}{r-1}-\frac{1}{r+1}$	A1	
	$S=\sum_{r=2}^{n}[\frac{1}{r-1}-\frac{1}{r+1}]$	B1	
	$S=[\frac{1}{1}-\frac{1}{3}]+$ $[\frac{1}{2}-\frac{1}{4}]+$ $[\frac{1}{3}-\frac{1}{5}]+$ $[\frac{1}{4}-\frac{1}{6}]+$ ... + $[\frac{1}{n-4}-\frac{1}{n-2}]+$ $[\frac{1}{n-3}-\frac{1}{n-1}]+$ $[\frac{1}{n-2}-\frac{1}{n}]+$ $[\frac{1}{n-1}-\frac{1}{n+1}]$	M1	
	$S=\frac{3}{2}-\frac{1}{n}-\frac{1}{n+1}$	A1	
	$\lim_{n\to\infty}S=\lim_{n\to\infty}[\frac{3}{2}-\frac{1}{n}-\frac{1}{n+1}]=\frac{3}{2}-0-0=\frac{3}{2}$	M1	
	S converges when n is large	A1	

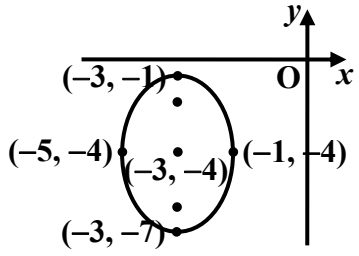
3(a)	$\mathbf{A}+\mathbf{B}=\begin{pmatrix}2&1&0\\-1&4&1\\3&0&-1\end{pmatrix}+\begin{pmatrix}-3&-1&2\\1&-1&0\\1&1&-1\end{pmatrix}=\begin{pmatrix}-1&0&2\\0&3&1\\4&1&-2\end{pmatrix}$	B1	5
	$(\mathbf{A}+\mathbf{B})\mathbf{C}=\begin{pmatrix}-1&0&2\\0&3&1\\4&1&-2\end{pmatrix}\begin{pmatrix}7&-2&6\\-4&6&-1\\12&-1&3\end{pmatrix}$	M1	
	$=\begin{pmatrix}17&0&0\\0&17&0\\0&0&17\end{pmatrix}$	A1	
	$(\mathbf{A}+\mathbf{B})^{-1}=\frac{1}{17}\mathbf{C}$	M1	
	$=\begin{pmatrix}\frac{7}{17}&-\frac{2}{17}&\frac{6}{17}\\-\frac{4}{17}&\frac{6}{17}&-\frac{1}{17}\\\frac{12}{17}&-\frac{1}{17}&\frac{3}{17}\end{pmatrix}$	A1	

3(b)	$x-2z=-17 \Rightarrow -x+2z=17$	B1	5
	$\begin{pmatrix}-1&0&2\\0&3&1\\4&1&-2\end{pmatrix}\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}17\\34\\17\end{pmatrix}$ $(\mathbf{A}+\mathbf{B})X=\mathbf{D}$	M1	
	$X=(\mathbf{A}+\mathbf{B})^{-1}\mathbf{D}$ $\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}\frac{7}{17}&-\frac{2}{17}&\frac{6}{17}\\-\frac{4}{17}&\frac{6}{17}&-\frac{1}{17}\\\frac{12}{17}&-\frac{1}{17}&\frac{3}{17}\end{pmatrix}\begin{pmatrix}17\\34\\17\end{pmatrix}$	M1	
	$\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}9\\7\\13\end{pmatrix}$	A1	
	$x=9, y=7, z=13$	A1	

4(a)	$u = \dfrac{5}{a+2i} \times \dfrac{a-2i}{a-2i}$	M1	2
	$= \dfrac{5a-10i}{a^2+4}$		
	$= \dfrac{5a}{a^2+4} - \dfrac{10}{a^2+4}i$	A1	

4(b)	$u^* = \dfrac{5a}{a^2+4} + \dfrac{10}{a^2+4}i$	B1	5
	$\text{Arg}(u^*) = \dfrac{3\pi}{4}$ $\tan^{-1}\left(\dfrac{\frac{10}{a^2+4}}{\frac{5a}{a^2+4}}\right) = \dfrac{3\pi}{4}$	M1	
	$\left(\dfrac{\frac{10}{a^2+4}}{\frac{5a}{a^2+4}}\right) = \tan\dfrac{3\pi}{4}$	M1	
	$\dfrac{2}{a} = \tan\dfrac{3\pi}{4}$	A1	
	$a = -2$	A1	

5(a)	$9(x^2 + 6x) + 4(y^2 + 8y) + 109 = 0$	M1	4
	$9\,(x+3)^2 + 4\,(y+4)^2 = 36$	M1	
	$\dfrac{(x+3)^2}{4} + \dfrac{(y+4)^2}{9} = 1$	A1	
	Conic is an ellipse	B1	

	Centre = (–3, –4)	B1	
	Vertices = (–3 , –1) and (–3, –7)	B1	
	Foci = $(-3, -4+\sqrt{5})$ and $(-3, -4-\sqrt{5})$	B1	
		All labels correct. D1 Correct shape of vertical ellipse D1	

8(a)	$\frac{z_2}{z_1} = \frac{(3-i)(1+2i)}{(1-2i)(1+2i)}$	M1	9
	$= \frac{3+6i-i+2}{5}$	M1	
	$=1+i$	A1	
	$\left\|\frac{z_2}{z_1}\right\| = \sqrt{2}$	B1	
	$\arg\left(\frac{z_2}{z_1}\right) = \frac{\pi}{4}$	B1	
	$\frac{z_2}{z_1} = \sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$	B1	
	$\left(\frac{z_2}{z_1}\right)^{20} = \left(\sqrt{2}\right)^{20}\left[\cos\ 20\left(\frac{\pi}{4}\right) + i\sin\ 20\left(\frac{\pi}{4}\right)\right]$	M1	
	$= 2^{10}(\cos 5\pi + i\sin 5\pi)$	A1	
	$= -1024$	A1	

8(b)	(i) $z - i = 2\cos\theta + i - 2i\sin\theta - i$		6
	$= 2\cos\theta - 2i\sin\theta$	B1	
	$\|z-i\| = \sqrt{(2\cos\theta)^2 + (-2\sin\theta)^2}$	M1	
	$\|z-i\| = 2$	A1	
	(ii) $\frac{1}{z+2-i} = \frac{1}{(2\cos\theta+2) - 2i\sin\theta} \times \frac{(2\cos\theta+2) + 2i\sin\theta}{(2\cos\theta+2) + 2i\sin\theta}$	M1	
	$= \frac{(2\cos\theta+2) + 2i\sin\theta}{(2\cos\theta+2)^2 + 4\sin^2\theta}$		
	For real part, $= \frac{2\cos\theta+2}{(2\cos\theta+2)^2 + 4\sin^2\theta}$	M1	
	$= \frac{2\cos\theta+2}{4\cos^2\theta + 8\cos\theta + 4 + 4\sin^2\theta}$		
	$= \frac{2\cos\theta+2}{8+8\cos\theta}$		
	$= \frac{1}{4}$	A1	

TAMAT SAMPEL

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