SAMPEL PERCUMA

Mock Test STPM 2020 Mathematics T Semester 1 Sarawak

Set penuh: Kertas Percubaan STPM

Sem 1 · Mathematics T

Sampel ini: 8 muka surat. Set penuh ada:

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- Blueprint 30 hari Semester 1 (setiap subjek)
- Semester 3: percubaan 2024, 10 subjek, banyak negeri, soalan + skema dalam satu fail

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2020-1-SAR-MOCT TEST

Section A [*45 marks*]

Answer **all** *questions.*

1. The functions *g* and *h* are defined as follows:

$$g: x \to e^x - 1,\ x \in R$$
$$h: x \to \ln ln\ (lnx)\ ,\ x > 1$$

(a) Explain why the composite function *hg* does not exist. [1]

(b) By restricting the domain of g to (k, ∞), where $k \in R$, find the smallest value of k in exact form such that the composite function *hg* exists. Define *hg*. [4]

(c) Solve the equation $hg(x) = \ln 5$, exactly. [2]

2.(a) Given that $S_n = \sum_{r=1}^{n} \frac{(r+1)^2}{2^r} = 11 - \frac{n^2+6n+11}{2^n}$ for all positive integers n, show that

$$\sum_{r=1}^{n} \frac{(r+2)^2}{2^r} = 18 - \frac{n^2+8n+18}{2^n}$$ [3]

(b) By considering the partial fractions of the expression $\frac{3}{r(r-1)(r-2)}$, show that

$$\sum_{r=3}^{N} \frac{3}{r(r-1)(r-2)} < \frac{3}{4}$$ [5]

3. A matrix C is given by $C = (1\ 2\ 1\ 1\ -1\ 2\ 1\ 2\ -1)$. By using elementary row operations, find the inverse of C. Hence, solve the equation $C(x\ y\ z) = (1\ 2\ 3)$. [5] [2]

4. (a) The complex number w is given by $w = \frac{\cos cos\ \theta + isin\ \theta}{1 - (\cos cos\ \theta + isin\ \theta)}$, where $0 < \theta < 2\pi$.

(i) Find the real part of w. [2]

(ii) Show that the imaginary part of w is $\frac{1}{2}cot\left(\frac{1}{2}\theta\right)$. [2]

(b) Find all the roots of the equation $z^4 - 64i = 0$, giving each root in the form $r(\cos\ \theta + i\sin\ \theta)$, where $r > 0$ and $-\pi < \theta \leq \pi$. [4]

5. The parametric equations of a conic are $x = 2(sec\ \beta + 1)$ and $y = 2tan\ \beta$, where $0 \leq \beta \leq 2\pi$.

(a) Find the standard form for the equation of the conic and identify the conic. [3]

(b) State the coordinates of the centre and the foci of the conic. [3]

6. The planes π_1 and π_2 have equations $3x + 2y + 4z = 6$ and $y = 0$ respectively.
(a) Find the acute angle between the planes π_1 and π_2. [3]
(b) Find the coordinates of the point P, the foot of the perpendicular from the origin O to the plane π_1. Hence, show that the perpendicular distance from the origin O to the plane π_1 is $\frac{6}{\sqrt{29}}$. [6]

Section B[15 *marks*]
Answer **one** *question only.*
You may answer all the questions **but***, only the first answer will be marked.*

7.(a) Show that $tan\ 4\theta = \frac{4t(1-t^2)}{1-6t^2+t^4}$ where $t = tan\ \theta$. [2]

Deduce that , when $tan\ \theta = \frac{1}{5}$, $tan\left(4\theta - \frac{1}{4}\pi\right) = \frac{1}{239}$ [3]

(b) Solve the trigonometric equation $3\sin 2\theta - \text{cosec } 2\theta + 2 = 0$ for $0^0 \le \theta \le 360^0$. [3]

(c) Express $\lambda\, sin\theta + (1-\lambda)\, cos\ \theta$ in the form of $Rsin(\theta + \phi)$, where R(R > 0) and tan ϕ are to be given in terms of λ. [3]

Write down an expression in terms of λ, for the minimum value of $\lambda\, sin\theta + (1-\lambda)\, cos\ \theta$ as θ varies, show that , for all λ, this minimum is less than or equal to $-\frac{1}{2}\sqrt{2}$. [4]

8(a) The lines l_1 and l_2 have equations $\mathbf{r} = (5 - \lambda)\mathbf{i} + 2\mathbf{j} + 3\lambda\mathbf{k}$ and $\frac{x-4}{-2} = \frac{y}{3} - 1 = -z$ respectively. Determine if the lines l_1 and l_2 intersect. Hence, state whether the lines are skew lines. [4]
(b) The position vectors of the points P, Q and R with respect to an origin O are respectively given by

$\mathbf{p} = 3\mathbf{i}$, $\mathbf{q} = 2\mathbf{j}$, and $\mathbf{r} = 9\mathbf{k}$.

(i) Find the exact value of the cosine of angle PRQ. [4]
(ii) Calculate the area of the triangle PRQ. [3]
(iii) Show that the volume of the tetrahedron OPQR is $\frac{1}{6}\left|\vec{OP} \cdot \left(\vec{PQ} \times \vec{PR}\right)\right|$, and calculate its value.

[4]

2020-1-SAR-MOCK TEST-MARKING SCHEME

1a	Range of g, $R_g = (-1, \infty)$ Domain of h, $D_h = (1, \infty)$ Since $R_g \not\subseteq D_h$, the composite function hg does not exist	M1
b	The smallest value of k such that the condition $R_g \subseteq D_h$ is satisfied occurs when $e^x - 1 > 1$ $e^x > 2 \quad x > ln\ 2$ the smallest value of $k = ln\ 2$ $hg(x) = h(e^x - 1) = \ln ln\left(ln(e^x - 1)\right)$ $hg: x \rightarrow \ln ln\left(ln(e^x - 1)\right),\ x > ln2$	M1 A1 M1 A1
c	$\ln ln\left(ln(e^x - 1)\right) = \ln 5$ $ln(e^x - 1) = 5$ $e^x - 1 = e^5$ $e^x = e^5 + 1$ $x = \ln(e^5 + 1)$	M1 A1

2a)	By replacing r by r – 1 $\sum_{r=1}^{n} \frac{(r+2)^2}{2^r} = \sum_{r=2}^{n+1} \frac{(r+1)^2}{2^{r-1}} = 2\sum_{r=2}^{n+1} \frac{(r+1)^2}{2^r}$ $= 2\left[\sum_{r=1}^{n+1} \frac{(r+1)^2}{2^r} - \frac{(1+1)^2}{2^1}\right]$ $= 2\left[11 - \frac{(n+1)^2+6(n+1)+11}{2^{n+1}} - 2\right]$ $= 2\left[9 - \frac{n^2+2n+1+6n+6+11}{2^{n+1}}\right]$ $= 18 - \frac{n^2+8n+18}{2^n}$	M1 M1 M1
b)	$\frac{3}{r(r-1)(r-2)} \equiv \frac{A}{r} + \frac{B}{r-1} + \frac{C}{r-2}$ $3 \equiv A(r-1)(r-2) + Br(r-2) + Cr(r-1)$ $r = 1,\ B = -3$ $r = 2,\ C = \frac{3}{2}$ $r = 0,\ A = \frac{3}{2}$	B1 A1

	$\frac{3}{r(r-1)(r-2)} \equiv \frac{3}{2r} - \frac{3}{r-1} + \frac{3}{2(r-2)}$ $\sum_{r=3}^{N} \frac{3}{r(r-1)(r-2)} = 3\sum_{r=3}^{N} \left[\frac{1}{2r} - \frac{1}{r-1} + \frac{1}{2(r-2)}\right]$ $= 3(\frac{1}{6} - \frac{1}{2} + \frac{1}{2}$ $+ \frac{1}{8} - \frac{1}{3} + \frac{1}{4}$ $+ \frac{1}{10} - \frac{1}{4} + \frac{1}{6}$ $+ \frac{1}{12} - \frac{1}{5} + \frac{1}{8}$ $+ \ldots.$ $+ \frac{1}{2(N-2)} - \frac{1}{N-3} + \frac{1}{2(N-4)}$ $+ \frac{1}{2(N-1)} - \frac{1}{N-2} + \frac{1}{2(N-3)}$ $+ \frac{1}{2N} - \frac{1}{N-1} + \frac{1}{2(N-1)}$ $= 3\lceil\frac{1}{4} + \frac{1}{2(N-1)} - \frac{1}{N-1} + \frac{1}{2N}\rceil$ $= \frac{3}{4} + 3\left(\frac{1}{2(N-1)} - \frac{1}{N-1} + \frac{1}{2N}\right)$ $= \frac{3}{4} + 3\left(\frac{N-2N+(N-1)}{2N(N-1)}\right)$ $= \frac{3}{4} - \frac{3}{2N(N-1)}$ Since $\frac{3}{2N(N-1)} > 0 \; for \; N \geq 3$, $\sum_{r=3}^{N} \frac{3}{r(r-1)(r-2)} < \frac{3}{4}$	At least 3 terms M1 At least 2 terms A1 M1

3	The augmented matrix is $(C \mid I) = (1\ 2\ 1\ 1\ -1\ 2\ 1\ 2\ -1 \mid 1\ 0\ 0\ 0\ 1\ 0\ 0\ 0\ 1)$ $-R_1 + R_2 \rightarrow R_2 \rightarrow (1\ 2\ 1\ 0\ -3\ 1\ 1\ 2\ -1 \mid 1\ 0\ 0\ -1$ $-R_1 + R_3 \rightarrow R_3 \rightarrow (1\ 2\ 1\ 0\ -3\ 1\ 0\ 0\ -2 \mid 1\ 0\ 0\ -1$ $-\frac{1}{3}R_3 \rightarrow R_2 \rightarrow \left(1\ 2\ 1\ 0\ 1\ -\frac{1}{3}\ 0\ 0\ -2 \mid 1\ 0\ 0\ \frac{1}{3}\ -\frac{1}{3}\right.$ $-\frac{1}{2}R_3 \rightarrow R_3 \rightarrow \left(1\ 0\ 0\ \frac{1}{3}\ -\frac{1}{3}\ 0\ \frac{1}{2}\ 0\ -\frac{1}{2}\right)$ $\frac{1}{3}R_3 + R_2 \rightarrow R_2 \rightarrow \left(1\ 0\ 0\ \frac{1}{2}\ -\frac{1}{3}\ -\frac{1}{6}\ \frac{1}{2}\ 0\ -\frac{1}{2}\right)$ $-R_3 + R_1 \rightarrow R_1 \rightarrow \left(\frac{1}{2}\ 0\ \frac{1}{2}\ \frac{1}{2}\ -\frac{1}{3}\ -\frac{1}{6}\ \frac{1}{2}\ 0\ -\frac{1}{2}\right)$ $-2R_2 + R_1 \rightarrow R_1 \rightarrow \left(-\frac{1}{2}\ \frac{2}{3}\ \frac{5}{6}\ \frac{1}{2}\ -\frac{1}{3}\ -\frac{1}{6}\ \frac{1}{2}\ 0\ -\right.$ $C^{-1} = \left(-\frac{1}{2}\ \frac{2}{3}\ \frac{5}{6}\ \frac{1}{2}\ -\frac{1}{3}\ -\frac{1}{6}\ \frac{1}{2}\ 0\ -\frac{1}{2}\right)$	B1 M1 Get first 2 “0” in any column & correct working

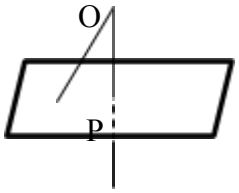
		M1 Get any 2 zero in any column M1 Identity matrix A1 Note: If no proper ERO or without arrow signs, NWW-1 If no proper form of matrix(without bracket or partition line), NWW-1
	C$(x\ y\ z) =$ $(1\ 2\ 3)$ $(x\ y\ z) =$ $C^{-1}(1\ 2\ 3)$ = $\left(-\frac{1}{2}\ \frac{2}{3}\ \frac{5}{6}\ \frac{1}{2}\ -\frac{1}{3}\ -\frac{1}{6}\ \frac{1}{2}\ 0\ -\frac{1}{2}\right)(1\ 2\ 3)$ =$\left(\frac{10}{3}\ -\frac{2}{3}\ -1\right)$ x=$\frac{10}{3}$, y= $-\frac{2}{3}$ z= -1	M1 A1

4a	$w = \frac{\cos\cos\theta+i\sin\theta}{1-(\cos\cos\theta+i\sin\theta)}$ = $\frac{\cos\cos\theta+i\sin\theta}{1-(\cos\cos\theta+i\sin\theta)}\ x\ \frac{\theta+i\sin\theta}{1-\cos\cos\theta+i\sin\theta}$ The real part of w = $\frac{\cos\cos\theta\,(1-\cos\theta)-\sin^2\theta}{(1-\cos\theta)^2+\sin^2\theta} = \frac{\cos\theta-1}{1-2\cos\theta+\cos^2\theta+\sin^2\theta}$ $=\frac{\cos\theta-1}{2(1-\cos\theta)} = -\frac{1}{2}$ the imaginary part of w = $\frac{\sin\theta\cos\theta+\sin\theta(1-\cos\theta)}{2(1-\cos\theta)} = \frac{\sin\theta}{2(1-\cos\theta)}$ $=\frac{2\sin\frac{1}{2}\theta\cos\frac{1}{2}\theta}{2\left(1-\left(1-2\sin^2\frac{1}{2}\theta\right)\right)} = \frac{\cos\frac{1}{2}\theta}{2\sin\frac{1}{2}\theta} = \frac{1}{2}\cot\left(\frac{1}{2}\theta\right)$	M1 conjugate A1 M1 A1
b	$z^4 = 64\left(\cos\cos\frac{\pi}{2} + i\sin\sin\frac{\pi}{2}\right)$ $z = \left[64\left(\cos\cos\left(\frac{\pi}{2}+2k\pi\right) + i\sin\sin\left(\frac{\pi}{2}+2k\pi\right)\right)\right]^{\frac{1}{4}}$, k =0,1,2,3 $=\sqrt[4]{64}\left(\cos\cos\left(\frac{\pi}{8}+\frac{2k\pi}{4}\right) + i\sin\sin\left(\frac{\pi}{8}+\frac{2k\pi}{4}\right)\right)$, k =0,1,2,3	B1 B1 M1

	$= \{2\sqrt{2}\left(\cos cos\left(\frac{\pi}{8}\right) + i \sin sin\left(\frac{\pi}{18}\right)\right),\ k = 0\ 2\sqrt{2}\left(\cos cos\right.$ $= \{2\sqrt{2}\left(\cos cos\ \frac{\pi}{8} + i \sin sin\ \frac{\pi}{8}\right),\ k = 0\ 2\sqrt{2}\left(\cos cos\ \frac{5\pi}{8}\right.$ $= \{2\sqrt{2}\left(\cos cos\ \frac{\pi}{8} + i \sin sin\ \frac{\pi}{8}\right),\ k = 0\ 2\sqrt{2}\left(\cos cos\ \frac{5\pi}{8}\right.$	A1, must show k =0,1,2,3

5a	$sec\,\beta = \frac{x-2}{2}$ and $tan\,\beta = \frac{y}{2}$ $1 + tan^2\,\beta = sec^2\,\beta$ $1 + \frac{y^2}{2^2} = \frac{(x-2)^2}{2^2}$ $\frac{(x-2)^2}{4} - \frac{y^2}{4} = 1$ Conic is a hyperbola	M1 sub tan and sec A1 function A1
b	Centre = (2,0) $c^2 = a^2 + b^2 = 4 + 4 = 8$ $c = 2\sqrt{2}$ Foci are $(2+ 2\sqrt{2}, 0)$ and $(2 - 2\sqrt{2}, 0)$	B1 M1 A1

6a	$3x + 2y + 4z = 6$ $(x\ y\ z).(3\ 2\ 4) = 6 \quad r.(3\ 2\ 4) = 6$ $y=0$ $(x\ y\ z).(0\ 1\ 0) = 0 \quad r.(0\ 1\ 0) = 0$ $cos\theta = \frac{\lvert(3\ 2\ 4).(0\ 1\ 0)\rvert}{\sqrt{(3)^2+(2)^2+(4)^2}\sqrt{(0)^2+(1)^2+(0)^2}}$ $= \frac{\lvert 0+2+0\rvert}{\sqrt{29}\sqrt{1}}$	

	$= \frac{2}{\sqrt{29}}$ $\theta = 68.2^0$	M1 dot product of $(3\ 2\ 4\ )\ and (0\ 1\ 0$ M1 dot product operation and vector magnitudes A1
b	 $\vec{OP}$ // $(3\ 2\ 4\ )$ or equation of line OP, r=$(3t\ 2t\ 4\ t)$ $\vec{OP}$ = t$(3\ 2\ 4\ )$, for some t $\in R$ P lies in the plane π_1 $\vec{OP}.(3\ 2\ 4\ ) = 6$ Sub $t = \frac{6}{29}$ into $\vec{OP}$ =$(3t\ 2t\ 4\ t)$ $\vec{OP}$ =$\left(3(\frac{6}{29})\ 2(\frac{6}{29})\ 4\ (\frac{6}{29})\right)$= $\left(\frac{18}{29}\ \frac{12}{29}\ \frac{24}{29}\right)$ Hence, the coordinates of P are ($\frac{18}{29}$, $\frac{12}{29}$, $\frac{24}{29}$) Perpendicular distance from the origin O to the plane π_1=OP $= \left\vert\vec{OP}\right\vert$ $=\sqrt{\left(\frac{18}{29}\right)^2 + \left(\frac{12}{29}\right)^2 + \left(\frac{24}{29}\right)^2}$ $=\frac{1}{29}\sqrt{1044}$ $=\frac{6\sqrt{29}}{29} = \frac{6}{\sqrt{29}}$ 9t+4t+16t =6	A1 M1 A1 A1 M1 A1

7a	$tan\ 4\theta = \frac{2\tan tan\ 2\theta}{1-tan^2 2\theta}$ $= \frac{2\left[\frac{2tan\theta}{1-tan^2\theta}\right]}{1-\left[\frac{2tan\theta}{1-tan^2\theta}\right]^2} = \frac{2\left[\frac{2t}{1-t^2}\right]}{1-\left[\frac{2t}{1-t^2}\right]^2} = \frac{4t(1-t^2)}{1-6t^2+t^4}$ $tan\ \theta = \frac{1}{5} = t$	M1 A1 M1

	$tan\,4\theta = \frac{4\left(\frac{1}{5}\right)\left(1-\left(\frac{1}{5}\right)^2\right)}{1-6\left(\frac{1}{5}\right)^2+\left(\frac{1}{5}\right)^4} = \frac{120}{119}$ $tan\left(4\theta - \frac{1}{4}\pi\right) = \frac{tan4\theta - \tan tan\frac{1}{4}\pi}{1+tan4\theta \tan tan\frac{1}{4}\pi} = \frac{\frac{120}{119}-1}{1+\left(\frac{120}{119}\right)(1)} = \frac{1}{239}$	M1 A1
7b	$3\sin 2\theta - \operatorname{cosec} 2\theta + 2 = 0$ $3\sin 2\theta - \frac{1}{sin2\theta} + 2 = 0$ $3\,sin^2 2\theta + 2sin2\theta - 1 = 0$ $(\sin 2\theta + 1)(3\sin 2\theta - 1) = 0$ $\sin 2\theta = -1 \qquad \sin 2\theta = \frac{1}{3}$ $2\theta = 270^0, 630^0 \qquad 2\theta = 19.47^0, 160.53^0, 379.47^0, 520.53^0$ $\theta = 135^0, 315^0 \qquad \theta = 9.74^0, 80.27^0, 189.74^0, 260.27^0$ $9.74^0, 80.27^0, 135^0, 189.74^0, 260.27^0, 315^0$	M1 A1 A1
7c	$\lambda\,sin\theta + (1-\lambda)cos\,\theta = Rsin(\theta+\phi)$ $= R\sin\theta cos\phi + Rcos\,\theta sin\phi$ $R\cos\phi = \lambda \qquad R\sin\phi = 1-\lambda$ $\tan\phi = \frac{1-\lambda}{\lambda}$ $R = \sqrt{2\lambda^2 - 2\lambda + 1}$ $\lambda\,sin\theta + (1-\lambda)cos\,\theta = \sqrt{2\lambda^2 - 2\lambda + 1}\,sin(\theta+\phi)$	B1 both correct M1 solve A1
	$-1 \le sin(\theta+\phi) \le 1$ $-\sqrt{2\lambda^2 - 2\lambda + 1} \le \sqrt{2\lambda^2 - 2\lambda + 1}\,sin(\theta+\phi) \le \sqrt{2\lambda^2 -$ $\min = -\sqrt{2\lambda^2 - 2\lambda + 1}$ $= -\sqrt{2\left[\lambda^2 - \lambda\right] + 1}$ $= -\sqrt{2\left[\lambda^2 - \lambda + \left(\frac{-1}{2}\right)^2 - \left(-\frac{1}{2}\right)^2\right] + 1}$	B1 M1 completing the squares

TAMAT SAMPEL

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