SAMPEL PERCUMA

Trial STPM 2021 Mathematics T Semester 1 Kedah (Keat Hwa, Alor Setar)

Set penuh: Kertas Percubaan STPM

Sem 1 · Mathematics T

Sampel ini: 7 muka surat. Set penuh ada:

- 3 fail PDF
- Semester 1: kertas percubaan (trial) negeri, subjek yang dipilih + skema
- Blueprint 30 hari Semester 1 (setiap subjek)
- Semester 3: percubaan 2024, 10 subjek, banyak negeri, soalan + skema dalam satu fail

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KEAT HWA SECONDARY SCHOOL

Alor Setar, Kedah

1 Feb 2021

MATHEMATICS T

PAPER 1 (954/1)

SIJIL TINGGI PERSEKOLAHAN MALAYSIA (STPM)

(MALAYSIA HIGHER SCHOOL CERTIFICATE)

TRIAL EXAM 2021

One and a half hour

DO NOT OPEN THE QUESTION PAPER UNTIL YOU ARE TOLD TO DO SO

Instructions to Candidates

1. Answer **all** questions in Section A
2. Answer **one** question only in Section B.
3. Write the answers on the answer sheets provided. Begin each answer on a new page of the answer sheet. All workings should be shown.
4. Scientific calculators may be used. Programmable and graphic display calculators are **not** allowed.
5. Answers may be written either in English or Malay.

Your informations

Name : _____

Class : _____

Section A [45 marks]

Answer **all** questions in this section.

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1. (a) (i) Find the largest subset of $\mathbb{R}$ such that $f(x) = \ln(2 - x)$ defines a function $f : S \longrightarrow \mathbb{R}$.

[1 mark]

(ii) Find an expression for $f^{-1}(x)$ and state its domain and range.

[4 marks]

(iii) Sketch, in a single diagram, the graph of $y = f(x)$ and $y = f^{-1}(x)$, making clear the relationship between the graphs.

[2 marks]

(b) If the coefficient of x in the expansion of $(4 + x)^n$ is equal to the coefficient of x^3 in the expansion $(4 + x)^{n+1}$ where n is a positive integer, find the value of n.

With this value of n, find the expansion of $(4 + x)^n$.

[5 marks]

2. (a) A geometric progression G has positive first term a, a common ratio r and sum to infinity S. The sum to infinity of the even-numbered terms of G, i.e. the second, fourth, sixth, ... terms, is $-\frac{1}{2}S$.

(i) Find the value of r.

[3 marks]

(ii) Given that the third term of G is 2, show that the sum to infinity of the odd-numbered terms of G is $\frac{81}{4}$.

[2 marks]

(iii) In another geometric progression H, each term is the modulus of the corresponding term of G. Show that the sum to infinity of H is $2S$.

[2 marks]

(b) Prove that

$$\sum_{r=2}^{n} \ln\left(1 - \frac{1}{r^2}\right) = \ln\left(\frac{n+1}{2n}\right).$$

[3 marks]

3. Given that

$$A = \begin{pmatrix} 1 & a & 2 \\ 2 & 1 & 2 \\ 2 & 2 & b \end{pmatrix},$$

where a and b are constants. Given also $A^2 - 4A = 5I$, where I is a 3×3 matrix. Show that $a = 2$ and $b = 1$. Hence, find A^{-1}.

[5 marks]

4. (a) Given that the complex number z_1 and its conjugate satisfy the equation

$$z_1 + 2z_1^* = (3 + 4i)^2.$$

Find z_1.

[3 marks]

(b) If $z_2 = 3 - i\sqrt{3}$, find the modulus and the argument of $\dfrac{8z_2}{z_2^*}$.

[4 marks]

5. Find the equation of the hyperbola (in standard form) with asymptotes $3y + 4x - 10 = 0$ and $3y - 4x + 2 = 0$, and hyperbola passing the point $(4, 2)$. Sketch the hyperbola and indicate its centre and vertices.

[6 marks]

6. If the lines $l_1 : \mathbf{r} = \begin{pmatrix} 1 \\ -5 \\ 7 \end{pmatrix} + \lambda \begin{pmatrix} a-1 \\ -a-1 \\ b \end{pmatrix}$ and $l_2 : \mathbf{r} = \begin{pmatrix} 9 \\ 3 \\ 8 \end{pmatrix} + \mu \begin{pmatrix} 2a \\ 3-5a \\ 15 \end{pmatrix}$ are parallel.

Find the values of a and b.

[5 marks]

Section B [15 marks]

Answer **one** question only in this section.

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7. A curve C is given parametrically by the equations $x = 2 + t$ and $y = 1 - t^2$.

(a) Show that the normal at the point with parameter t has equation

$$x - 2ty = 2t^3 - t + 2.$$

[4 marks]

(b) The normal at point T, where $t = 2$, cuts C again at point P, where $t = p$. Find the coordinates of point P.

[7 marks]

(c) Find the Cartesian equation of C and sketch C.

[4 marks]

8. (a) The points A and B have position vectors $2\mathbf{i} - 3\mathbf{j} + 2\mathbf{k}$ and $5\mathbf{i} - 2\mathbf{j} + \mathbf{k}$ respectively. The plane p has equation $x + y = 5$.

(i) Find the position vector of the point of the intersection of the line through A and B and the plane p.

[4 marks]

(ii) A second plane q has an equation of the form $x + by + cz = d$, where b, c and d are constants. The plane q contains the line AB, and the acute angle between p and q is $60°$. Find the equation of q.

[7 marks]

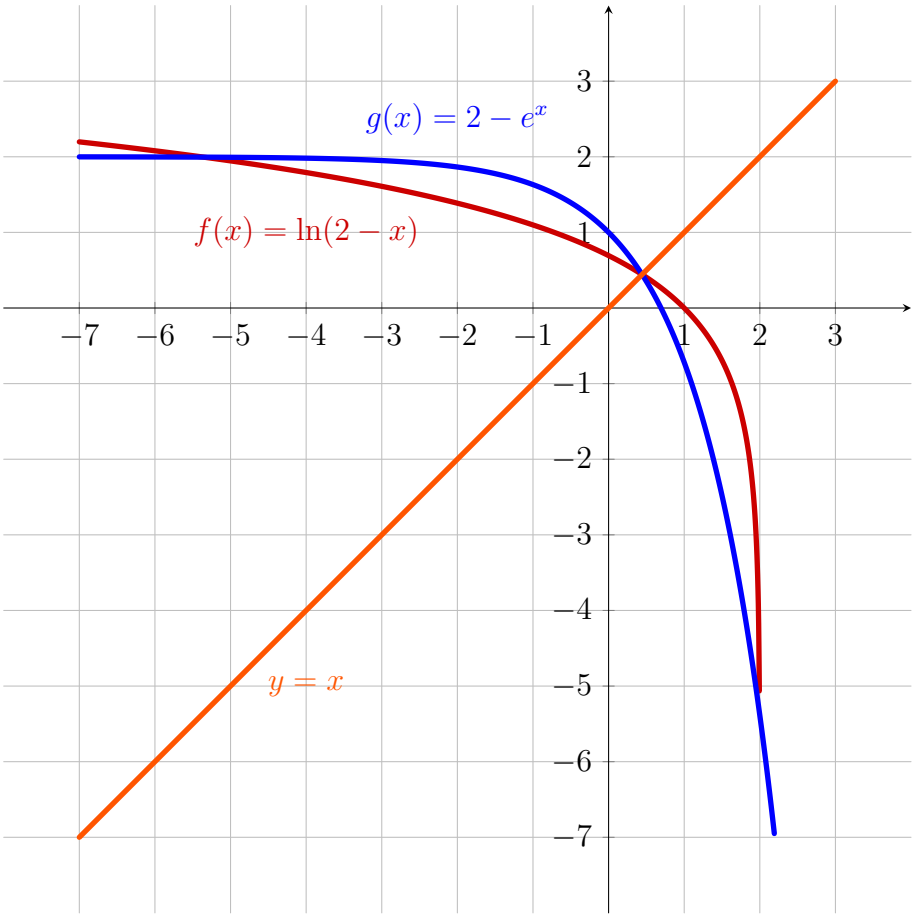
(b) Show that

$$\frac{1}{3 + \sqrt{5}} \leq \frac{1}{3 + \sin\theta - 2\cos\theta} \leq \frac{1}{3 - \sqrt{5}}.$$

[4 marks]

END OF QUESTIONS

GOOD LUCK



(b) For $(4+x)^n$, the term contains x is the 2nd term in the expansion of $(4+x)^n$. Hence, the coefficient of x in the expansion of $(4+x)^n$ is

$$\binom{n}{1} 4^{n-1}$$

For $(4+x)^{n+1}$, the term contains x^3 is the 4th term in the expansion of $(4+x)^{n+1}$. Hence, the coefficient of x^3 in the expansion of $(4+x)^{n+1}$ is

$$\binom{n+1}{3} 4^{n+1-3} = \binom{n+1}{3} 4^{n-2}$$

Given coefficient of x in expansion of $(4+x)^n$ = coefficient of x^3 in expansion of $(4+x)^{n+1}$.

$$\Rightarrow \binom{n}{1} 4^{n-1} = \binom{n+1}{3} 4^{n-2}$$

$$\Rightarrow \left(\frac{n!}{1!(n-1)!}\right)(4^n)\left(\frac{1}{4}\right) = \left(\frac{(n+1)!}{3!(n+1-3)!}\right)(4^n)\left(\frac{1}{16}\right)$$

$$\Rightarrow \left(\frac{n!}{(n-1)(n-2)!}\right)\left(\frac{1}{4}\right) = \left(\frac{(n+1)(n!)}{6(n-2)!}\right)\left(\frac{1}{16}\right)$$

$$n^2 = 25$$

$$n = \pm 5$$

$$\Rightarrow \begin{pmatrix} 1+2a & 4-2a & 2a-2b-6 \\ 0 & 2a+1 & 2b-2 \\ 2b-2 & 2a-2b-6 & 8-4b+b^2 \end{pmatrix} = \begin{pmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{pmatrix}$$

Comparing:

$$1+2a = 5 \;\Rightarrow a = 2$$
$$2b-2 = 0 \;\Rightarrow b = 1$$

To deduce A^{-1}: Use back $A^2 - 4A = 5I$:

$$A^2 - 4A = 5I$$
$$\Rightarrow A^{-1}A^2 - 4A^{-1}A = 5A^{-1}I$$
$$A - 4I = 5A^{-1}$$
$$\Rightarrow A^{-1} = \frac{1}{5}A - \frac{4}{5}I$$
$$\Rightarrow A^{-1} = \frac{1}{5}\begin{pmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{pmatrix} + \frac{4}{5}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} -\frac{3}{5} & \frac{2}{5} & \frac{2}{5} \\ \frac{2}{5} & -\frac{3}{5} & \frac{2}{5} \\ \frac{2}{5} & \frac{2}{5} & -\frac{3}{5} \end{pmatrix} = \begin{pmatrix} -0.6 & 0.4 & 0.4 \\ 0.4 & -0.6 & 0.4 \\ 0.4 & 0.4 & -0.6 \end{pmatrix}$$

4. (a) Solving $z_1 + 2z_1^* = (3+4i)^2$.

Let $z_1 = a + bi$, thus, $z_1^* = a - bi$. Substitute back into the equation:

$$z_1 + 2z_1^* = (3+4i)^2$$
$$\Rightarrow (a+bi) + 2(a-bi) = 9 + 24i + 16i^2$$
$$\Rightarrow 3a - bi = -7 + 24i$$

Comparing:

$$3a = -7 \;\Rightarrow a = -\frac{7}{3}$$
$$-b = 24 \;\Rightarrow b = -24$$

Hence, $z_1 = -\frac{7}{3} - 24i$.

(b) Given $z_2 = 3 - i\sqrt{3}$, thus $z_2^* = 3 + i\sqrt{3}$.

$$\Rightarrow \frac{8z_2}{z_2^*} = \frac{8(3-i\sqrt{3})}{3+i\sqrt{3}}$$
$$\Rightarrow = 8\left(\frac{(3-i\sqrt{3})(3-i\sqrt{3})}{(3+i\sqrt{3})(3-i\sqrt{3})}\right) = 8\left(\frac{9-6i\sqrt{3}+3i^2}{12}\right) = 4 - 4\sqrt{3}i$$

Modulus of $\frac{8z_2}{z_2^*}$, $\left|\frac{8z_2}{z_2^*}\right| = \sqrt{(4)^2 + (-4\sqrt{3})^2} = 8$.

6. Given the lines $l_1 : \mathbf{r} = \begin{pmatrix} 1 \\ -5 \\ 7 \end{pmatrix} + \lambda \begin{pmatrix} a-1 \\ -a-1 \\ b \end{pmatrix}$ and $l_2 : \mathbf{r} = \begin{pmatrix} 9 \\ 3 \\ 8 \end{pmatrix} + \mu \begin{pmatrix} 2a \\ 3-5a \\ 15 \end{pmatrix}$ are parallel.

Let the direction vector of line l_z be $\mathbf{a} = \begin{pmatrix} a-1 \\ -a-1 \\ b \end{pmatrix}$ and that of line l_2 be $\mathbf{b} = \begin{pmatrix} 2a \\ 3-5a \\ 15 \end{pmatrix}$.

Since both lines are parallel, we can express the relationship between $\mathbf{a}$ and $\mathbf{b}$ as:

$$\mathbf{a} = t\mathbf{b}$$

where t is a constant.

$$\Rightarrow \begin{pmatrix} a-1 \\ -a-1 \\ b \end{pmatrix} = t \begin{pmatrix} 2a \\ 3-5a \\ 15 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} a-1 \\ -a-1 \\ b \end{pmatrix} = \begin{pmatrix} 2at \\ 3t-5at \\ 15t \end{pmatrix}$$

Comparing:

$$a - 1 = 2at \tag{5}$$

$$-a - 1 = 3t - 5at \tag{6}$$

$$b = 15t \tag{7}$$

From (5): $t = \dfrac{a-1}{2a}$.

Substitute into (6):

$$-a - 1 = 3\left(\frac{a-1}{2a}\right) - 5a\left(\frac{a-1}{2a}\right)$$

$$3a^2 - 10a + 3 = 0$$

$$(3a-1)(a-3) = 0$$

Hence, $a = \dfrac{1}{3}$ or $a = 3$.

From (5):

When $a = \dfrac{1}{3}$:

$$\frac{1}{3} - 1 = 2\left(\frac{1}{3}\right)t \;\Rightarrow\; t = -1$$

When $a = 3$:

$$3 - 1 = 2(3)t \;\Rightarrow\; t = \frac{1}{3}$$

From (7):

When $t = -1$:

$$b = 15(-1) = -15$$

When $t = \dfrac{1}{3}$:

$$b = 15\left(\frac{1}{3}\right) = 5$$

TAMAT SAMPEL

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