SAMPEL PERCUMA

Trial STPM 2021 Mathematics T Semester 1 Kelantan

Set penuh: Kertas Percubaan STPM

Sem 1 · Mathematics T

Sampel ini: 7 muka surat. Set penuh ada:

- 3 fail PDF
- Semester 1: kertas percubaan (trial) negeri, subjek yang dipilih + skema
- Blueprint 30 hari Semester 1 (setiap subjek)
- Semester 3: percubaan 2024, 10 subjek, banyak negeri, soalan + skema dalam satu fail

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Sampel bermula di muka surat sebelah

Section A (45 marks)

Answer **all** the questions in this section.

1. Functions $f(x)$ and $g(x)$, each defined for $-1 < x < 1$, are given by $f(x) = \ln(1-x)$ and $g(x) = x^2$.
   (i) Find $f^{-1}(x)$ and state its domain and range. [4 marks]

   (ii) Show that $f(x) + f(-x) = f \circ g$. [3 marks]

2. Find $\sum_{r=1}^{n}(1-2r)^2$ in term of n. Give your answer in a fully factorized form. Hence, evaluate [6 marks]

$$\sum_{r=25}^{75}(1-2r)^2$$

3. (a) Show that the determinant of the matrix

$$A = \begin{pmatrix} x^2 & x & 1 \\ y^2 & y & 1 \\ z^2 & z & 1 \end{pmatrix}$$

   is $(y-x)(z-x)(y-z)$.
   If $x = y = 2$ and $z\ \varepsilon\ \Re$, deduce that A is a singular matrix. [4 marks]

   (b) By substituting $x = 1, y = 2$ and $z = 3$, find the inverse of matrix A. [4 marks]

4. Express the complex numbers $\sqrt{3} + i$ and $2 - 2i$ in polar form.

   Determine $\left(\frac{\sqrt{3}+i}{2-2i}\right)^{10}$. Hence , solve the equation $z^3 = \left(\frac{\sqrt{3}+i}{2-2i}\right)^{10}$, giving each answer in polar form. [9 marks]

5. The conic section given by $9y^2 - 4x^2 + 18y + 16x - 43 = 0$
   (a) Express the equation in the standard form. [2marks]
   (b) Determine the coordinates of the centre, the vertices and the foci of the hyperbola. [5 marks]
   (c) Find the equations of the asymptotes and sketch the graph. [2 marks]

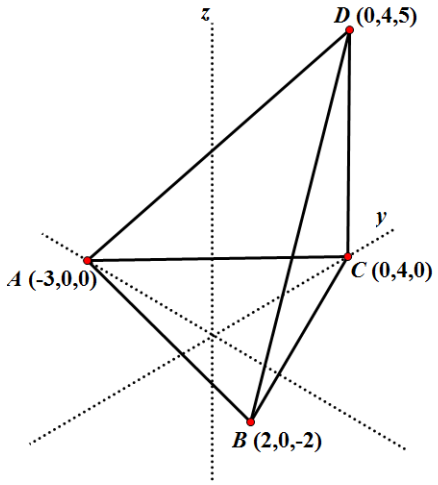
6. $\overrightarrow{OA}$ and $\overrightarrow{OB}$ are two vectors such that $\overrightarrow{OA} = \boldsymbol{a} + 2\boldsymbol{b}$, $\overrightarrow{OB} = 2\boldsymbol{a} - \boldsymbol{b}$ and $\overrightarrow{OA}$ and $\overrightarrow{OB}$ are mutually perpendicular. a and b are magnitudes of **a**, **b** respectively.
   (a) Show that $\mathbf{a}.\boldsymbol{b} = \frac{2}{3}b^2 - a^2$. [3 marks]
   (b) If $\boldsymbol{a}$ has magnitude 4 units and $\boldsymbol{b}$ has magnitude 5 units, find the angle between $\boldsymbol{a}$ and $\boldsymbol{b}$. [3 marks]

Section B (15 marks)

Answer **only** one question from this section

7. Polynomial $\mathrm{P}(\mathrm{x}) = 2\mathrm{x}^3 + \mathrm{ax}^2 + \mathrm{bx} + 6$ is divisible by $(\mathrm{x} - 3)$ but leaves a remainder of -12 when it is divided by $(\mathrm{x} + 1)$.
   (a) Find the value of a and b. [4 marks]
   (b) Factorise $\mathrm{P}(\mathrm{x})$ completely. [2 marks]
   (c) Solve the equation $6\mathrm{x}^3 + \mathrm{bx}^2 + \mathrm{ax} + 2 = 0$. [4 marks]
   (d) Find the set of values of x such that P
   $$P(\mathrm{x}) < (\mathrm{x} - 2)(\mathrm{x} - 3)(\mathrm{x} + 3)$$ [5 marks]

8. Figure shows a tetrahedron.



   (i) Find the lengths of the edges AB and AC and the size of the angle CAB. Hence, calculate the area of triangle ABC.
   (ii) Show that $4i - 3j + 10k$ is normal to the plane ABC. Hence find the equation of this plane. [6 marks]
   (iii) Write down a vector equation for the line through D perpendicular to the plane ABC and find the point of intersection of this line with the plane ABC. [4 marks] [5 marks]

Marking Scheme

Section A (45 marks)
Answer **all** the questions in this section.

1. Functions $f(x)$ and $g(x)$, each defined for $-1 < x < 1$, are given by $f(x) = \ln(1-x)$ and $g(x) = x^2$.

(i) Find $f^{-1}(x)$ and state its domain and range. [4 marks]

(ii) Show that $f(x) + f(-x) = f \circ g$. [3 marks]

(i) $let\ f^{-1}(x) = y$

$f^{-1}(x) = y$

$\ln(1-y) = x$ **M1**

$y = 1 - e^x$

$\therefore f^{-1}(x) = 1 - e^x$ A1

$Domain\ f = \{x \epsilon \Re x < ln2\}$ A1

$range\ f = \{y \epsilon \Re, -1 < y < 1\}$ A1

(ii) $f(-x) = \ln(1+x)$

$f \circ g = \ln(1-x^2)$ M1

$f(x) + f(-x) = \ln(1-x) + \ln(1+x)$

$= \ln[(1-x)(1+x)]$ M1

$= \ln(1-x^2)$

$= f \circ g$

2. Find $\sum_{r=1}^{n}(1-2r)^2$ in term of n. Give your answer in a fully factorized form. Hence, evaluate [6 marks]

$$\sum_{r=25}^{75}(1-2r)^2$$

$$\sum_{r=1}^{n}(1-2r)^2 = \sum_{r=1}^{n}1 - 4\sum_{r=1}^{n}r + 4\sum_{r=1}^{n}r^2$$ M1

$$= n - 4\left(\frac{n(n+1)}{2}\right) + 4\left(\frac{1}{6}(n)(n+1)(2n+1)\right)$$ M1

$$= \frac{1}{3}n(3 - 6(n+1) + 2(n+1)(2n+1)$$ M1

$$= \frac{1}{3}n(2n-1)(2n+1)$$ A1

$$\sum_{r=25}^{75}(1-2r)^2 = \frac{1}{3}(75)(150-1)(150+1) - \frac{1}{3}(25)(50-1)(50+1)$$ M1

$$= 544051$$ A1

3. (a) Show that the determinant of the matrix

$$A = \begin{pmatrix} x^2 & x & 1 \\ y^2 & y & 1 \\ z^2 & z & 1 \end{pmatrix}$$

is $(y-x)(z-x)(y-z)$. If $x=y=2$ and $z\ \varepsilon\ \Re$, deduce that A is a singular matrix. (b) By substituting $x=1, y=2$ and $z=3$, find the inverse of matrix A .	[4 marks] [4 marks]
(a) $\lvert A\rvert = \begin{vmatrix} x^2 & x & 1 \\ y^2 & y & 1 \\ z^2 & z & 1 \end{vmatrix}$ $= \begin{vmatrix} x^2 & x & 1 \\ y^2-x^2 & y-x & 0 \\ z^2-x^2 & z-x & 0 \end{vmatrix}$ M1 $= \begin{vmatrix} y^2-x^2 & y-x \\ z^2-x^2 & z-x \end{vmatrix}$ M1 $= (y^2-x^2)(z-x) - (z^2-x^2)(y-x)$ M1 $= (y-x)(z-x)(y+x-z-x)$ A1 $= (y-x)(z-x)(y-z)(shown)$ (b) $A = \begin{pmatrix} 1 & 1 & 1 \\ 4 & 2 & 1 \\ 9 & 3 & 1 \end{pmatrix}$ $\lvert A\rvert = (2-1)(3-1)(2-3)$ A1 $= -2$ $adjoint\ A = \begin{pmatrix} \begin{vmatrix} 2 & 1 \\ 3 & 1 \end{vmatrix} & -\begin{vmatrix} 4 & 1 \\ 9 & 1 \end{vmatrix} & \begin{vmatrix} 4 & 2 \\ 9 & 3 \end{vmatrix} \\ -\begin{vmatrix} 1 & 1 \\ 3 & 1 \end{vmatrix} & \begin{vmatrix} 1 & 1 \\ 9 & 1 \end{vmatrix} & -\begin{vmatrix} 1 & 1 \\ 9 & 3 \end{vmatrix} \\ \begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix} & -\begin{vmatrix} 1 & 1 \\ 4 & 1 \end{vmatrix} & \begin{vmatrix} 1 & 1 \\ 4 & 2 \end{vmatrix} \end{pmatrix}^T$ M1 $= \begin{pmatrix} -1 & 5 & -6 \\ 2 & -8 & 6 \\ -1 & 3 & -2 \end{pmatrix}^T$ $= \begin{pmatrix} -1 & 2 & -1 \\ 5 & -8 & 3 \\ -6 & 6 & -2 \end{pmatrix}$ $A^{-1} = \frac{1}{\lvert A\rvert} adjoint\ A$ $= \frac{1}{-2}\begin{pmatrix} -1 & 2 & -1 \\ 5 & -8 & 3 \\ -6 & 6 & -2 \end{pmatrix}$ M1 $A^{-1} = \begin{pmatrix} \frac{1}{2} & -1 & \frac{1}{2} \\ -\frac{5}{2} & 4 & -\frac{3}{2} \\ 3 & -3 & 1 \end{pmatrix}$ A1	
4. Express the complex numbers $\sqrt{3}+i$ and $2-2i$ in polar form. Determine $\left(\frac{\sqrt{3}+i}{2-2i}\right)^{10}$. Hence , solve the equation $z^3 = \left(\frac{\sqrt{3}+i}{2-2i}\right)^{10}$, giving each answer in polar form.	[9 marks]

$z_1 = 2[\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}]$ B1

$z_2 = 2[\sqrt{2}\cos(-\frac{\pi}{4}) + i\sin(-\frac{\pi}{4})]$ B1

$$\left(\frac{\sqrt{3}+i}{2-2i}\right) = \left(\frac{2}{2\sqrt{2}}\right)\left[\cos\left(\frac{\pi}{6}-\left(-\frac{\pi}{4}\right)\right) + i\sin\left(\frac{\pi}{6}-\left(-\frac{\pi}{4}\right)\right)\right]$$

$$= \left(\frac{1}{\sqrt{2}}\right)\left[\cos\left(\frac{5\pi}{12}\right) + i\sin\left(\frac{5\pi}{12}\right)\right]$$ M1

$$\left(\frac{\sqrt{3}+i}{2-2i}\right)^{10} = \left(\frac{1}{\sqrt{2}}\right)^{10}\left[\cos 10\left(\frac{5\pi}{12}\right) + i\sin 10\left(\frac{5\pi}{12}\right)\right]$$ B1 for $\frac{1}{32}$

$$= \left(\frac{1}{32}\right)\left[\cos\left(\frac{25\pi}{6}\right) + i\sin\left(\frac{25\pi}{6}\right)\right]$$

$$z^3 = \left(\frac{1}{32}\right)\left[\cos\left(\frac{25\pi}{6}\right) + i\sin\left(\frac{25\pi}{6}\right)\right]$$ M1 must include $2\pi k$

$$z = \left(\frac{1}{32}\right)^{\frac{1}{3}}\left[\cos\left(\frac{25\pi}{6}+2\pi k\right) + i\sin\left(\frac{25\pi}{6}+2\pi k\right)\right]^{\frac{1}{3}}$$

$$= \left(\frac{1}{\sqrt[3]{32}}\right)\left[\cos\frac{1}{3}\left(\frac{25\pi}{6}+2\pi k\right) + i\sin\frac{1}{3}\left(\frac{25\pi}{6}+2\pi k\right)\right]$$

$k = 1,\ z_1 = \frac{1}{2^{5/3}}\left[\cos\frac{25\pi}{18} + i\sin\frac{25\pi}{18}\right]$ A1

$k = 2,\ z_2 = \frac{1}{2^{5/3}}\left[\cos\frac{37\pi}{18} + i\sin\frac{27\pi}{18}\right]$ A1

$k = 3,\ z_3 = \frac{1}{2^{5/3}}\left[\cos\frac{49\pi}{18} + i\sin\frac{49\pi}{18}\right]$ A1

5. The conic section given by $9y^2 - 4x^2 + 18y + 16x - 43 = 0$
   (a) Express the equation in the standard form. [2marks]
   (b) Determine the coordinates of the centre, the vertices and the foci of the hyperbola. [5 marks]
   (c) Find the equations of the asymptotes and sketch the graph. [2 marks]

(a) $9y^2 - 4x^2 + 18y + 16x - 43 = 0$

$9y^2 + 18y - 4(x^2 - 4x) - 43 = 0$

$9[(y+1)^2 - 1] - 4[(x-2)^2 - 4] - 43 = 0$ M1

$9(y+1)^2 - 4(x-2)^2 = 36$

$\frac{(y+1)^2}{4} - \frac{(x-2)^2}{9} = 1$ A1

(b) Centre $= (2, -1)$ A1

$c = \sqrt{4+9}$

B1

7. Polynomial $P(x) = 2x^3 + ax^2 + bx + 6$ is divisible by $(x - 3)$ but leaves a remainder of -12 when it is divided by $(x + 1)$.

(a) Find the value of a and b. [4 marks]

(b) Factorise $P(x)$ completely. [2 marks]

(c) Solve the equation $6x^3 + bx^2 + ax + 2 = 0$. [4 marks]

(d) Find the set of values of x such that P

$$P(x) < (x - 2)(x - 3)(x + 3)$$ [5 marks]

(a) $P(3) = 0$

$54 + 9a + 3b + 6 = 0$

$9a + 3b = -60$

$3a + b = -20 - - - (1)$ M1

$P(-1) = -12$

$-2 + a - b + 6 = 0$

$a - b = -4 - - - (2)$ M1

$(1) + (2)$: $4a = -24$

$a = -6$ M1

$\therefore b = -2$

Therefore $a = -6$ and $b = -2$ A1

(b) $P(x) = 2x^3 - 6x^2 - 2x + 6$

$= (x - 3)(2x^2 - 2)$

$= 2(x - 3)(x^2 - 1)$ M1

$= 2(x - 3)(x - 1)(x + 1)$ B1

(c) $6x^3 + bx^2 + ax + 2 = 0$

$6x^3 - 2x^2 - 6x + 2 = 0$

$3x^3 - x^2 - 3x + 1 = 0$

$(x - 1)(3x^2 + 2x - 1) = 0$ M1

$(x - 1)(3x - 1)(x + 1) = 0$ M1

$x = 1, x = \frac{1}{3}$ or $x = -1$

Hence the solution is $x = 1, x = \frac{1}{3}$ and $x = -1$ A1 A1

(d) $P(x) < (x - 2)(x - 3)(x + 3)$

$2(x - 3)(x - 1)(x + 1) < (x - 2)(x - 3)(x + 3)$

$2(x - 3)(x - 1)(x + 1) - (x - 2)(x - 3)(x + 3) < 0$ M1

$(x - 3)[2(x - 1)(x + 1) - (x - 2)(x + 3)] < 0$

$(x - 3)[2(x^2 - 1) - (x^2 + x - 6)] < 0$ M1

$(x - 3)[x^2 - x + 4] < 0$

$(x - 3)\left[\left(x - \frac{1}{2}\right)^2 + \frac{15}{4}\right] < 0$ M1

$since\ \left(x-\frac{1}{2}\right)^2 \geq 0, therefore\ \left(x-\frac{1}{2}\right)^2+\frac{15}{4}>0\ \forall x \in \mathbb{R}$ B1

$Hence, x-3<0 \Rightarrow x<3$
$\therefore$ *the solution set is* $\{x: x<3\}$ A1

(i) $\overrightarrow{AB}=\begin{pmatrix}-2\\-1\\-1\end{pmatrix}$ A1

$\overrightarrow{AC}\begin{pmatrix}-1\\-11\\3\end{pmatrix}$ A1

(ii) **a**. $\overrightarrow{AB}=\begin{pmatrix}2\\-1\\-3\end{pmatrix}\begin{pmatrix}-2\\-1\\-1\end{pmatrix}=-4+1+3=0$ M1

a. $\overrightarrow{AC}=\begin{pmatrix}2\\-1\\-3\end{pmatrix}\begin{pmatrix}-\\-11\\3\end{pmatrix}=-2+11-9=0$ M1

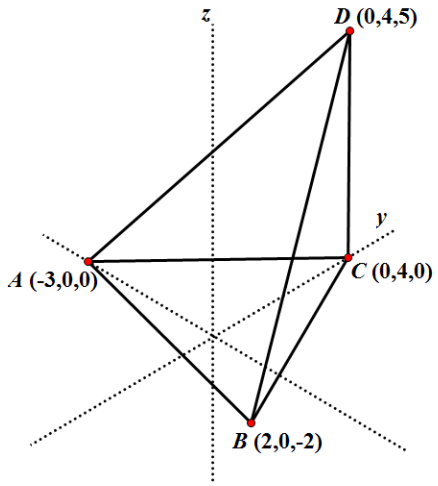
} **A1**

plane : $\boldsymbol{r}.\boldsymbol{n}=\boldsymbol{a}.\boldsymbol{n}$

$\begin{pmatrix}x\\y\\z\end{pmatrix}\begin{pmatrix}2\\-1\\-3\end{pmatrix}=\begin{pmatrix}1\\3\\-2\end{pmatrix}\begin{pmatrix}2\\-1\\-3\end{pmatrix}$ M1

$2x-y-3z=2-3+6$
$2x-y-3z=5$ A1

8. Figure shows a tetrahedron.



(i) Find the lengths of the edges AB and AC and the size of the angle CAB. Hence, calculate the area of triangle ABC. [6 marks]

(ii) Show that $4i-3j+10k$ is normal to the plane ABC. Hence find the equation of this plane. [4 marks]

(iii) Write down a vector equation for the line through D perpendicular to the plane ABC and find the point of intersection of this line with the plane ABC. [5 marks]

(i) $AB=\sqrt{5^2+(-2)^2}=\sqrt{29}$ A1

TAMAT SAMPEL

Suka sampel ini?

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