SAMPEL PERCUMA

Trial STPM 2021 Mathematics T Semester 1 Melaka

Set penuh: Kertas Percubaan STPM

Sem 1 · Mathematics T

Sampel ini: 7 muka surat. Set penuh ada:

- 3 fail PDF
- Semester 1: kertas percubaan (trial) negeri, subjek yang dipilih + skema
- Blueprint 30 hari Semester 1 (setiap subjek)
- Semester 3: percubaan 2024, 10 subjek, banyak negeri, soalan + skema dalam satu fail

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Section A [45 *marks*]

Answer **all** *questions.*

1. The function f is defined by $f(\mathrm{x}) = \frac{1}{2}(e^{x} - e^{-x})$, where $x \in \mathrm{R}$.

Find the inverse function of f, and state its domain. [7]

2. (a) Verify the identity $\frac{2r-1}{r(r-1)} - \frac{2r+1}{r(r+1)} \equiv \frac{2}{(r-1)(r+1)}$. [2]

(b) Hence, using the method of differences to prove that

$$\sum_{r=2}^{n} \frac{2}{(r-1)(r+1)} = \frac{3}{2} - \frac{2n+1}{n(n+1)}$$

[4]

(c) Deduce the sum of the infinite series

$$\frac{1}{1\times3} + \frac{1}{2\times4} + \frac{1}{3\times5} + \cdots + \frac{1}{(n-1)(n+1)} + \cdots$$ [2]

3. By using row elementary operation, solve this system of equations:

$$\begin{aligned} x - 2y + 3z &= 9 \\ -x + 3y &= -4 \\ 2x - 5y + 5z &= 17 \end{aligned}$$ [5]

4. The complex numbers z_1 and z_2 are given by $z_1 = 1 - 2i$ and $z_2 = 3 - i$. Find the modulus and argument of $\frac{z_2}{z_1}$. [5]

Hence, determine the real value of $\left(\frac{z_2}{z_1}\right)^{20}$. [4]

5. The parametric equations of a curve are $x = t - \frac{1}{t}$ and $y = t + \frac{1}{t}$. Find the Cartesian equation of the curve and state the curve. [3]

Find (a)The coordinates of the vertices and foci [2]
(b)The equations of the asymptotes [1]
(c) Sketch the curve [2]

6. Relative to an origin **O**, the position vectors of points **A** and **B** are $2\boldsymbol{i} + \boldsymbol{j} + 2\boldsymbol{k}$ and $3\boldsymbol{i} - 2\boldsymbol{j} + p\boldsymbol{k}$ respectively.

a) Find the value of p for which $\overrightarrow{OA}$ and $\overrightarrow{OB}$ are perpendicular. [3]
b) In the case where $p = 6$, use a scalar product to find angle ∠AOB. [5]

Section B [15 *marks*]

Answer **one** *question only.*

7. (a) Express $\frac{1}{(r^2-1)}$ in partial fractions, and deduce that $\frac{1}{r(r^2-1)} \equiv \frac{1}{2}\left[\frac{1}{r(r-1)} - \frac{1}{r(r+1)}\right]$ [4]

Hence, use the method of differences to find the sum of the first $(n-1)$ terms, S_{n-1}, of the series

$$\frac{1}{2\times3} + \frac{1}{3\times8} + \frac{1}{4\times15} + \cdots + \frac{1}{r(r^2-1)} + \cdots, \quad [6]$$

and deduce S_n.

(b) Explain why the series converges to $\frac{1}{4}$, determine the smallest values of n such that

$$\frac{1}{4} - S_n < 0.0025. \quad [5]$$

8. Given that $f(x) = 10\cos^2 x - 8\sin x\cos x + 4\sin^2 x$,

(a) express $f(x)$ in the form $a\cos 2x + b\sin 2x + c$, where a, b and c are constants.
If $f(x) = c + r\cos(2x+\alpha)$ where $r > 0$ and $0° < \alpha < 90°$, show that the greatest and least values of $f(x)$ are 12 and 2 respectively. [9]

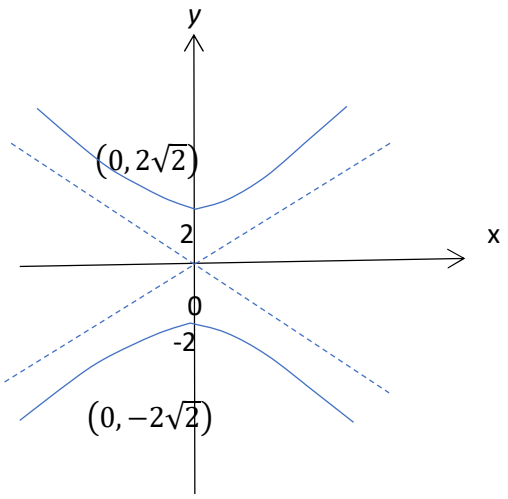
(b) solve the equation when $f(x) = 2$ for $0° < x < 360°$. [3]

(c) find the set of values of x such that $2 \le f(x) \le \frac{9}{2}$, in the interval $0° < x < 180°$. [3]

Scheme of Mark (Trial Semester 1 2021)

1	let f(x) = y $\frac{1}{2}(e^x - e^{-x}) = y$ $e^x - e^{-x} - 2y = 0$ $e^{2x} - 2ye^x - 1 = 0$ $e^x = \frac{2y \pm \sqrt{4y^2+4}}{2}$ $= y \pm \sqrt{y^2+1}$ Since $e^x > 0, \forall x \in \mathrm{R}$, then $e^x = y + \sqrt{y^2+1}$ $x = \ln(y + \sqrt{y^2+1})$ $f^{-1}: x \to \ln(x + \sqrt{x^2+1})$, $x \in \mathrm{R}$	B1(attempt to find inverse) M1 (quadratic equation) M1(using formula) M1(with reason choosing the positive range) M1(express in logarithmic function) M1A1(Correct inverse function in mapping notation with domain stated) [7 marks]
2a	$\frac{2r-1}{r(r-1)} - \frac{2r+1}{r(r+1)}$ $= \frac{(r+1)(2r-1) - (2r+1)(r-1)}{r(r-1)(r+1)}$ $= \frac{2r^2 - r + 2r - 1 - 2r^2 + 2r - r + 1}{r(r-1)(r+1)}$ $= \frac{2r}{r(r-1)(r+1)}$ $= \frac{2}{(r-1)(r+1)}$ [verified]	M1 equate the denominator and simply A1
b	$\sum_{r=2}^{n} \frac{2}{(r-1)(r+1)}$ $= \sum_{r=2}^{n} \left[\frac{2r-1}{r(r-1)} - \frac{2r+1}{r(r+1)}\right]$ $= \sum_{r=2}^{n} [f(r) - f(r+1)]$ $= f(2) - f(n+1)$ $= \frac{2(2)-1}{2(1)} - \frac{2(n+1)-1}{(n+1)(n+1-1)}$ $= \frac{3}{2} - \frac{2n+1}{n(n+1)}$	M1 M1 (method of difference) M1(sub value) A1

c	$\frac{1}{2}\sum_{r=2}^{\infty}\frac{2}{(r-1)(r+1)}$ $=\frac{1}{2}\lim_{n\to\infty}\left[\frac{3}{2}-\frac{2n+1}{n(n+1)}\right]$ $=\frac{1}{2}\lim_{n\to\infty}\left[\frac{3}{2}-\frac{\frac{2}{n}+\frac{1}{n^2}}{1+\frac{1}{n}}\right]$ $=\frac{1}{2}\left(\frac{3}{2}-0\right)=\frac{3}{4}$	M1 use the answer in part (b)A1 to find the limiting value A1 [8 marks]
3	$\left(\begin{array}{ccc\|c}1 & -2 & 3 & 9\\ -1 & 3 & 0 & -4\\ 2 & -5 & 5 & 17\end{array}\right)$ $\xrightarrow[R_3 \to -2R_1+R_3]{R_2 \to R_1+R_2}\left(\begin{array}{ccc\|c}1 & -2 & 3 & 9\\ 0 & 1 & 3 & 5\\ 0 & -1 & -1 & -1\end{array}\right)$ $\xrightarrow[R_3 \to R_2+R_3]{R_1 \to 2R_2+R_1}\left(\begin{array}{ccc\|c}1 & 0 & 9 & 19\\ 0 & 1 & 3 & 5\\ 0 & 0 & 2 & 4\end{array}\right)$ $\xrightarrow{R_3 \to \frac{1}{2}R_2}\left(\begin{array}{ccc\|c}1 & 0 & 9 & 19\\ 0 & 1 & 3 & 5\\ 0 & 0 & 1 & 2\end{array}\right)$ $\xrightarrow[R_2 \to -3R_3+R_2]{R_1 \to -9R_3+R_1}\left(\begin{array}{ccc\|c}1 & 0 & 0 & 1\\ 0 & 1 & 0 & -1\\ 0 & 0 & 1 & 2\end{array}\right)$ $x=1, y=-1, z=2$	B1 (augmented matrix) M1 → 0 at 2nd and 3rd row respective entries and at 2nd entry in the 1st row M1 → 1 M1 → 0 A1 [5]
4	$\frac{z_2}{z_1}=\frac{(3-i)(1+2i)}{(1-2i)(1+2i)}$ $=\frac{3+6i-i+2}{5}$ $=1+i$ $\left\|\frac{z_2}{z_1}\right\|=\sqrt{2}$ $\frac{z_2}{z_1}=\sqrt{2}\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)$ $\left(\frac{z_2}{z_1}\right)^{20}=\left(\sqrt{2}\right)^{20}\left[\cos\ 20\left(\frac{\pi}{4}\right)+i\sin 20\left(\frac{\pi}{4}\right)\right]$ $=2^{10}(\cos 5\pi+i\sin 5\pi)$ $=-1024$	M1 M1 A1 B1 B1 M1 A1 A1 [9]

5	$y^2 - x^2 = \left(t + \frac{1}{t}\right)^2 - \left(t - \frac{1}{t}\right)^2$ $= t^2 + 2 + \frac{1}{t^2} - t^2 + 2 - \frac{1}{t^2}$ $= 4$ $\frac{y^2}{4} - \frac{x^2}{4} = 1$ This is the standard form of a hyperbola	M1 A1 B1
	$a = 2, b = 2,\ c = \sqrt{a^2 + b^2} = \sqrt{2^2 + 2^2} = 2\sqrt{2}$ Vertices are (0, −2) and (0,2) Foci are $(0, -2\sqrt{2})\ and\ (0, 2\sqrt{2})$ Asymptotes are $y = \pm\frac{a}{b}x$ $y = x$ or $y = -x$	B1 vertices B1 foci A1 (both equations)
		D1 (Correct shape of hyperbola with vertical transverse axes) D1 (All correct, with points label – asymptotes pass through origin) [8]
6.	Perpendicular $\underset{\sim}{a}.\underset{\sim}{b} = 0$ i) $\begin{pmatrix}2\\1\\2\end{pmatrix} \cdot \begin{pmatrix}3\\-2\\p\end{pmatrix} = 0$ $6 - 2 + 2p = 0$ $2p = -4$ $p = -2$ ii) $p = 6$ $\overrightarrow{OA} = 2\underset{\sim}{i} + \underset{\sim}{j} + 2\underset{\sim}{k}\ \ \overrightarrow{OB} = 3\underset{\sim}{i} - 2\underset{\sim}{j} + 6\underset{\sim}{k}$ $\overrightarrow{OA}.\ \overrightarrow{OB} = \begin{pmatrix}2\\1\\2\end{pmatrix} \cdot \begin{pmatrix}3\\-2\\6\end{pmatrix}$ $= 6 - 2 + 12$ $= 16$ $\left\|\overrightarrow{OA}\right\| = \sqrt{2^2 + 1^2 + 2^2}$ or $\left\|\overrightarrow{OB}\right\| = \sqrt{3^2 + (-2)^2 + 6^2}$ $= \sqrt{9}$ $= \sqrt{49}$ $= 3$ $= 7$ $\text{Cos}\ \theta = \frac{\left\|\overrightarrow{OA}\cdot\overrightarrow{OB}\right\|}{\left\|\overrightarrow{OA}\right\|\left\|\overrightarrow{OB}\right\|}$ $= \frac{16}{21}$ $\theta = 40.37°$	B1 (dot product) M1(attempt to solve p) A1 M1 B1M1 M1 A1 [8]

7a	$\frac{1}{r^2-1} \equiv \frac{A}{r-1} + \frac{B}{r+1}$ $A + B = 0$ and $A - B = 1; A = \frac{1}{2}, B = -\frac{1}{2}$ $\frac{1}{r^2-1} \equiv \frac{1}{2}\left[\frac{1}{(r-1)} - \frac{1}{(r+1)}\right]$ $\frac{1}{r(r^2-1)} \equiv \frac{1}{2}\left[\frac{1}{r(r-1)} - \frac{1}{r(r+1)}\right]$ $S_{n-1} = \sum_{r=2}^{n} \frac{1}{2}\left[\frac{1}{r(r-1)} - \frac{1}{r(r+1)}\right]$ $= \sum_{r=2}^{n} \frac{1}{2}[f(r) - f(r+1)]$ $= \frac{1}{2}[f(2) - f(n+1)]$ $= \frac{1}{2}\left[\frac{1}{2} - \frac{1}{n(n+1)}\right]$ $S_n = \frac{1}{2}\left[\frac{1}{2} - \frac{1}{(n+1)(n+2)}\right]$	B1 partial fraction M1 correct equations and values of A and B A1 B1 (deduced) M1 M1 (method of difference) M1 (sub) A1 M1 A1(able to replace n with n+1)
7b	As $n \to \infty, \frac{1}{(n+1)(n+2)} \to 0$ The limiting value for S_n exists and equals to $\frac{1}{4}$, therefore the series converges to $\frac{1}{4}$ $\frac{1}{4} - S_n < 0.0025.$ $\frac{1}{4} - \frac{1}{2}\left[\frac{1}{2} - \frac{1}{(n+1)(n+2)}\right] < 0.0025.$ $\frac{1}{(n+1)(n+2)} < 0.005$ $(n+1)(n+2) > 200$ since n is positive, $n > 12.65$ The smallest n is 13.	B1 B1(conclusion) M1 M1 Solve inequalities (with reason n is positive and choose $n > 12.65$) A1 [15]
8a	$f(x) = 10\cos^2 x - 8\sin x\cos x + 4\sin^2 x$ $= 4\sin^2 x + 4\cos^2 x + 6\cos^2 x - 4\sin 2x$ $= 4 + 3(1 + \cos 2x) - 4\sin 2x$ $= 7 + 3\cos 2x - 4\sin 2x$ $f(x) = c + r\cos(2x + \alpha)$	B1 M1 A1

	$r\cos(2x+\alpha) = 3\cos 2x - 4\sin 2x$ $r[\cos 2x\cos\alpha - \sin 2x\sin\alpha] = 3\cos 2x - 4\sin 2x$ $r\cos\alpha = 3, \quad r\sin\alpha = 4$ $r^2 = 3^2 + 4^2 \Rightarrow r = 5,$ $\frac{\sin\alpha}{\cos\alpha} = \frac{4}{3} \Rightarrow \tan\alpha = \frac{4}{3} \Rightarrow \alpha = \tan^{-1}(\frac{4}{3}) \Rightarrow \alpha = 53.1°$ $f(x) = 7 + 3\cos 2x - 4\sin 2x = 7 + 5\cos(2x + 53.1°)$ $-1 \le \cos(2x+53.1°) \le 1$ $-5 \le 5\cos(2x+53.1°) \le 5$ $7-5 \le 7+5\cos(2x+53.1°) \le 7+5$ $2 \le 7+5\cos(2x+53.1°) \le 12$ Greatest and least values of $f(x)$ are 12 and 2 respectively.	B1(r correct) B1 (correct angle) A1 B1 A1 A1(both)
	(b) $f(x) = 7+5\cos(2x+53.1°) = 2$ $\Rightarrow f(x) = \cos(2x+53.1°) = -1$ $2x+53.1° = 180°, 540°$ $x = 63.5°, 243.5°$ (c) $2 \le f(x) \le \frac{9}{2} \Rightarrow 2 \le 7+5\cos(2x+53.1°) \le \frac{9}{2}$ $\Rightarrow -1 \le \cos(2x+53.1°) \le -\frac{1}{2}$ $\cos(2x+53.1°) = -1, \quad \cos(2x+53.1°) = -\frac{1}{2}$ $2x+53.1° = 180°, \quad 2x+53.1° = 120°, 240°$ $x = 63.5°, \quad x = 33.5°, 93.5°$ $x = \{x : 33.5° \le x \le 93.5°\}$	B1 B1 A1 B1 M1 (solve the x values) A1

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