SAMPEL PERCUMA

Trial STPM 2021 Mathematics T Semester 1 Negeri Sembilan (SMK KGV)

Set penuh: Kertas Percubaan STPM

Sem 1 · Mathematics T

Sampel ini: 7 muka surat. Set penuh ada:

- 3 fail PDF
- Semester 1: kertas percubaan (trial) negeri, subjek yang dipilih + skema
- Blueprint 30 hari Semester 1 (setiap subjek)
- Semester 3: percubaan 2024, 10 subjek, banyak negeri, soalan + skema dalam satu fail

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Section A [45 *marks*]

Answer **all** *questions.*

1 The function f and g are defined by

$$f: x \to x^2 - 1,\ x \in R$$

$$g: x \to \frac{x+1}{x-1},\ x \in R,\ x \neq 1$$

Find the composite function fg, and state its domain and range. [5]

2 The 1st and 2nd, 3rd and 4th, 5th and 6th,... terms of a G.P. are added up to form a new series. Show that the new series is also a G.P. and find S_n. [7]

3 Let $M = \begin{pmatrix} p & -1 & -2 \\ 1 & 1 & -2 \\ 1 & q & -1 \end{pmatrix}$ and $M^{-1} = \frac{1}{2}\begin{pmatrix} 3 & -5 & 4 \\ -1 & 1 & 0 \\ 1 & -3 & 2 \end{pmatrix}$

Find the value of p and of q.

Hence, solve the system of linear equations.

$$px - y - 2z = 7$$
$$x + y - 2z = 2$$
$$x + qy - z = -3$$ [8]

4 Given that the complex number z and its conjugate z* satisfy the equation $zz^* + 2zi = 12 + 6i$. Find the possible values of z. [8]

5 The parametric equations of a curve are $x = t - \frac{1}{t}$ and $y = t + \frac{1}{t}$. By taking $y^2 - x^2$ or otherwise, show that the curve is a hyperbola. [3]

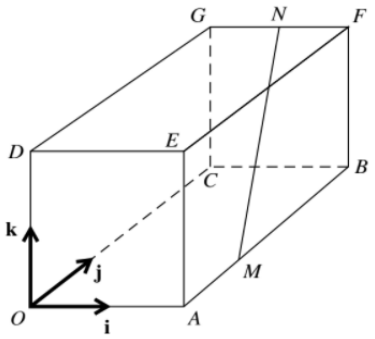
Find

(a) the coordinates of the vertices and foci. [3]

(b) the equation of the asymptotes. [1]

Sketch the curve. [2]

6



In the diagram above, OABCDEFG is a cuboid in which OA = 2 units, OC = 3 units and OD = 2 units. Unit vectors **i**, **j** and **k** are parallel to OA, OC and OD respectively. The point M on AB is such that MB = 2AM. The midpoint of FG is N.

(a) Express the vectors $\overrightarrow{OM}$ and $\overrightarrow{MN}$ in terms of **i**, **j** and **k**. [3]

(b) Find a vector equation for the line through M and N. [1]

(c) Find the position vector of P, the foot of the perpendicular from D to the line through M and N. [4]

Section B [15 *marks*]
Answer **one** *question only.*

7 (a) Find the set of values of x such that $-2 < x^3 - 2x^2 + x - 2 < 0$ [7]

(b) Sketch, on the same coordinate axes, the graphs of $y = |x - 3|$, $x \in R$ and $y^2 = x + 3$, $x \geq -3$.

Hence, find the set of values of x such that $|x - 3| > \sqrt{x+3}$. [8]

8 Consider the points A(1, 0, 0), B(2, 2, 2) and C(0, 2, 1).

(a) Find the vector equation of the line L_1 passing through the points A and C. [2]

(b) Find the vector $\overrightarrow{CA} \times \overrightarrow{CB}$. [3]

(c) Find an exact value for the area of the triangle ABC. [2]

(d) Show that the Cartesian equation of the plane π_1, containing the triangle ABC, is $2x + 3y - 4z = 2$. [3]

A second plane π_2 is defined by the Cartesian equation π_2: $4x - y - z = 4$. L_2 is the line of intersection of the planes π_1 and π_2.

(e) Find a vector equation of L_2. [5]

SULIT*

MATHEMATICAL FORMULAE

Trigonometrical identities

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B + \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Sums of series

For an arithmetic series

$$S_n = \frac{1}{2}n(a + \ell) = \frac{1}{2}n[2a + (n-1)d]$$

For a geometric series

$$S_n = \frac{a(1 - r^n)}{1 - r}, \; a \neq 1$$

Binomial expansions

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n, \quad n \in \mathbb{Z}^+$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots + \frac{n(n-1)\cdots(n-r+1)}{r!}x^r + \ldots, \quad n \in \mathbb{Q}, \; |x| < 1$$

Conics

Parabola with vertex (h, k), focus $(a + h, k)$ and directrix $x = -a + h$

$$(y-k)^2 = 4a(x-h)$$

Ellipse with centre (h, k) and foci $(-c + h, k)$, $(c + h, k)$, where $c^2 = a^2 - b^2$

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

Hyperbola with centre (h, k) and foci $(-c + h, k)$, $(c + h, k)$, where $c^2 = a^2 + b^2$

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$

Solution

1 $fg(x)$

$= f\left(\frac{x+1}{x-1}\right)$ B1

$= \left(\frac{x+1}{x-1}\right)^2 - 1$ M1

$= \frac{x^2+2x+1-(x^2-2x+1)}{(x-1)^2}$

$= \frac{4x}{(x-1)^2}, x \in R, x \neq 1$

$fg: x \to \frac{4x}{(x-1)^2}, x \in R, x \neq 1$ A1

Domain of $fg = \{x : x \in R, x \neq 1\}$ A1

Range of $fg = \{y : y \geq -1\}$ A1 [5]

2 Let the given G.P. has 1st term = a and common ratio = r

Then, for the new series

1st term, $U_1 = a + ar = a(1 + r)$ M1

2nd term, $U_2 = ar^2 + ar^3 = ar^2(1 + r)$ M1

3rd term, $U_3 = ar^4 + ar^5 = ar^4(1 + r)$

and so on

$\frac{U_n}{U_{n-1}} = \frac{ar^{2(n-1)}(1+r)}{ar^{2(n-2)}(1+r)}$ M1

$= r^2\ (constant)$ A1

∴The new series is also a G.P. with 1st term = a (1 + r) & common ratio = r^2 A1

$S_n = \frac{a(1+r)[(r^2)^n-1]}{r^2-1}$ M1

$= \frac{a(1+r)[r^{2n}-1]}{(r+1)(r-1)}$

$= \frac{a[r^{2n}-1]}{r-1}$ A1 [7]

3 $MM^{-1} = \begin{pmatrix} p & -1 & -2 \\ 1 & 1 & -2 \\ 1 & q & -1 \end{pmatrix} \frac{1}{2} \begin{pmatrix} 3 & -5 & 4 \\ -1 & 1 & 0 \\ 1 & -3 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

$\frac{1}{2}(3p + 1 - 2) = 1$ or any equivalents M1

$3p = 3$

$p = 1$ A1

$\frac{1}{2}(3 - q - 1) = 0$ or any equivalents M1

$q = 2$ A1

$$\begin{pmatrix}1 & -1 & -2\\ 1 & 1 & -2\\ 1 & 2 & -1\end{pmatrix}\begin{pmatrix}x\\ y\\ z\end{pmatrix}=\begin{pmatrix}7\\ 2\\ -3\end{pmatrix}$$ B1

$$\begin{pmatrix}x\\ y\\ z\end{pmatrix}=\frac{1}{2}\begin{pmatrix}3 & -5 & 4\\ -1 & 1 & 0\\ 1 & -3 & 2\end{pmatrix}\begin{pmatrix}7\\ 2\\ -3\end{pmatrix}$$ M1

$$=\frac{1}{2}\begin{pmatrix}-1\\ -5\\ -5\end{pmatrix}$$

$$=\begin{pmatrix}-0.5\\ -2.5\\ -2.5\end{pmatrix}$$ A1

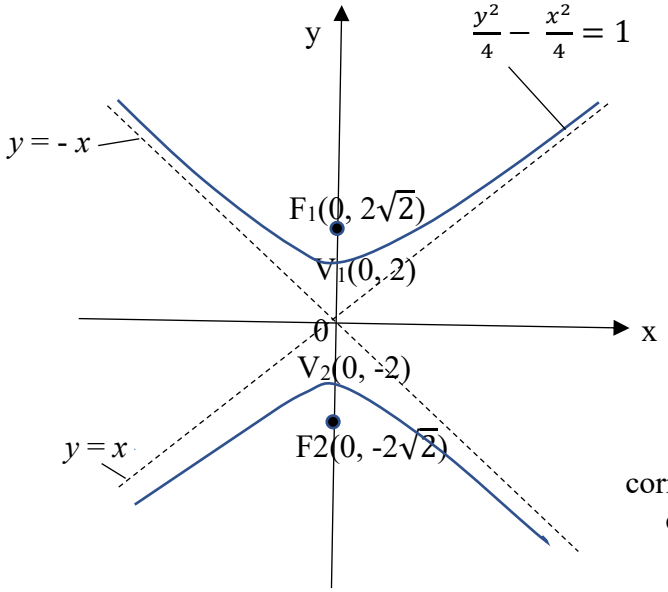
x = -0.5, y = -2.5, z = -2.5 A1 [8]

4 Let $z = x + yi$
and $z^* = x - yi$ B1
Given $zz^* + 2zi = 12 + 6i$
$(x + yi)(x - yi) + 2i(x + yi) = 12 + 6i$ M1
$x^2 + y^2 - 2y + 2xi = 12 + 6i$ M1
Equating coefficient of imaginary parts, $2x = 6$ — either one M1
$x = 3$ A1
Equating the real parts, $x^2 + y^2 - 2y = 12$

$y^2 - 2y - 3 = 0$ M1

$(y + 1)(y - 3) = 0$
$y = -1$ or 3 A1
$z = 3 + 3i$ or $3 - i$ A1 [8]

5 $y^2 - x^2 = (t + \frac{1}{t})^2 - (t - \frac{1}{t})^2$
$y^2 - x^2 = t^2 + 2 + \frac{1}{t^2} - t^2 + 2 - \frac{1}{t^2}$ M1
$y^2 - x^2 = 4$
$\frac{y^2}{4} - \frac{x^2}{4} = 1$ A1
It is a hyperbola with vertical axis and centre (0, 0). A1
(a) Vertices are (0, -2) and (0, 2) A1
$c = \sqrt{4 + 4} = 2\sqrt{2}$ M1
Foci are $(0, -2\sqrt{2})$, $(0, 2\sqrt{2})$ A1
(b) Asymptotes are $y = -x$ and $y = x$ A1



correct shape & points D1
correct asymptotes D1 [9]

6 (a) $\overrightarrow{OM} = \overrightarrow{OA} + \overrightarrow{AM}$
$= 2\mathbf{i} + \mathbf{j}$ A1
$\overrightarrow{MN} = \overrightarrow{MB} + \overrightarrow{BF} + \overrightarrow{FN}$ M1
$= -\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$ A1

(b) Vector equation for the line through M and N, $\mathbf{r} = 2\mathbf{i} + \mathbf{j} + \lambda(-\mathbf{i} + 2\mathbf{j} + 2\mathbf{k})$ A1

(c) $\overrightarrow{DP} = \overrightarrow{OP} - \overrightarrow{OD}$
$= (2 - \lambda)\mathbf{i} + (1 + 2\lambda)\mathbf{j} + 2\lambda\mathbf{k} - 2\mathbf{k}$
$= (2 - \lambda)\mathbf{i} + (1 + 2\lambda)\mathbf{j} + (-2 + 2\lambda)\mathbf{k}$ B1

$\overrightarrow{DP} \cdot \overrightarrow{MN} = 0$

$\begin{pmatrix} 2 - \lambda \\ 1 + 2\lambda \\ -2 + 2\lambda \end{pmatrix} \cdot \begin{pmatrix} -1 \\ 2 \\ 2 \end{pmatrix} = 0$ M1

$-2 + \lambda + 2 + 4\lambda - 4 + 4\lambda = 0$

$\lambda = \frac{4}{9}$ A1

The position vector of P $= (2 - \frac{4}{9})\,\mathbf{i} + [1 + 2(\frac{4}{9})]\mathbf{j} + 2(\frac{4}{9})\,\mathbf{k}$

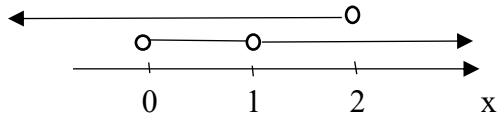
$= \frac{14}{9}\mathbf{i} + \frac{17}{9}\mathbf{j} + \frac{8}{9}\mathbf{k}$ A1 [8]

7 (a) $0 < x^3 - 2x^2 + x$ and $x^3 - 2x^2 + x - 2 < 0$ M1

$x(x^2 - 2x + 1) > 0$ $x^2(x - 2) + x - 2 < 0$

$x(x-1)^2 > 0$ $(x-2)(x^2 + 1) < 0$ M1M1

$x > 0, x \neq 1$ $x < 2$ A1A1



M1

Set of values of x $= \{x: 0 < x < 2, x \neq 1\}$ A1 [7]

(d) $\mathbf{r} \cdot \mathbf{n} = \mathbf{a} \cdot \mathbf{n}$

$\mathbf{r} \cdot \begin{pmatrix} -2 \\ -3 \\ 4 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} -2 \\ -3 \\ 4 \end{pmatrix}$ M1

$-2x - 3y + 4z = -2 + 0 + 0$ M1

$2x + 3y - 4z = 2$ A1

(e) A vector which is parallel to the line of intersection between plane π_1 and π_2

$= \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ -2 & -3 & 4 \\ 4 & -1 & -1 \end{vmatrix}$ M1

$= \begin{vmatrix} -3 & 4 \\ -1 & -1 \end{vmatrix} \boldsymbol{i} - \begin{vmatrix} -2 & 4 \\ 4 & -1 \end{vmatrix} \boldsymbol{j} + \begin{vmatrix} -2 & -3 \\ 4 & -1 \end{vmatrix} \boldsymbol{k}$

$= 7\,\boldsymbol{i} + 14\boldsymbol{j} + 14\boldsymbol{k}$ A1

$= 7\,(\boldsymbol{i} + 2\boldsymbol{j} + 2\boldsymbol{k})$

To find a point on the line L_2:

$2x + 3y - 4z = 2$

$4x - y - z = 4$

Let $z = 0$, $2x + 3y = 2$ --------(1) attempt to solve M1

$4x - y = 4$ ---------(2)

$2(1) - (2)$, $7y = 0$

$y = 0$

$x = 1$

$\therefore (1, 0, 0)$ is a point on L_2. A1

The vector equation for L_2 is $\boldsymbol{r} = \boldsymbol{i} + \mu(\boldsymbol{i} + 2\boldsymbol{j} + 2\boldsymbol{k})$ A1 [15]

TAMAT SAMPEL

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- Blueprint 30 hari Semester 1 (setiap subjek)
- Semester 3: percubaan 2024, 10 subjek, banyak negeri, soalan + skema dalam satu fail

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