SAMPEL PERCUMA

Trial STPM 2021 Mathematics T Semester 1, SMK St. Paul Seremban

Set penuh: Kertas Percubaan STPM

Sem 1 · Mathematics T

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2021-1-NS-SPI

Section A [45 marks]

Answer **all** questions in this section.

1. The polynomial P(x) is given by P(x) = $x^3 + x^2 - 10x + 8$
   a) Using the factor theorem, show that $x - 2$ is a factor of P(x). [2]
   b) Hence express P(x) as the product of three linear factors. [3]

2. a) Expand $\left(r + \frac{1}{2}\right)^4$ using binomial expansions. [2]
   b) Verify that $4r^3 + r \equiv \left(r + \frac{1}{2}\right)^4 - \left(r - \frac{1}{2}\right)^4$ [3]
   c) Hence find $\sum_{r=1}^{n}(4r^3 + r)$ [4]

3. Given the matrices $A = \begin{pmatrix} -1 & 2 & 1 \\ -3 & 1 & 4 \\ 0 & 1 & 2 \end{pmatrix}$, $B = \begin{pmatrix} -35 & 19 & 18 \\ -27 & -13 & 45 \\ -3 & 12 & 5 \end{pmatrix}$
   and $A^2B = \begin{pmatrix} 121 & 0 & 0 \\ 0 & 121 & 0 \\ 0 & 0 & 121 \end{pmatrix}$

   a) Deduce the inverse of A. [3]

   b) Hence, solve the system of linear equations

   $$x - 2y - z = -8$$
   $$3x - y - 4z = -15$$
   $$y + 2z = 4$$

   [5]

4. a) By using de Moivre's theorem, show that $\cos 5\theta = \cos\theta(16\cos^4\theta - 20\cos^2\theta + 5)$. [4]
   b) Hence, show that $x = \cos\left(\frac{1}{10}\pi\right)$ is a root of the equation $16x^4 - 20x^2 + 5 = 0$. [4]

5. The parametric equations of a curve are $x = 1 + a\cos\theta$, $y = b\sin\theta - 2$.
   a) Show that the cartesian equation of the conics is $\frac{(x-1)^2}{a^2} + \frac{(y+2)^2}{b^2} = 1$. [2]
   b) States the coordinates of its centre and foci when a = 2 and b = 3. [2]
   Hence, sketch the conics. [3]

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6. The line l_1 has equation $\underline{r_1} = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$ and

the Line l_2 has equation $\underline{r_2} = \begin{pmatrix} 1 \\ 3 \\ 6 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}$

a) Show that l_1 and l_2 do not meet. [3]

The point A is on l_1 where $\lambda = 1$, and the point B is on l_2 where $\mu = 2$.

b) Find the cosine of the acute angle between AB and l_1. [5]

Section B [15 marks]

Answer **one** question only in this section.

7. a) Using the identity prove that $\cos 2A \equiv 1 - 2\sin^2 A$. [2]

b) Show that $2\sin 2\theta - 3\cos 2\theta - 3\sin\theta + 3 \equiv \sin\theta(4\cos\theta + 6\sin\theta - 3)$. [3]

c) Express $4\cos\theta + 6\sin\theta$ in the form $R\sin(\theta + \alpha)$, where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$.

[4]

d) Hence, for $0 < \theta < \pi$, solve $2\sin 2\theta = 3(\cos 2\theta + \sin\theta - 1)$,

giving your answers in radians to 3 decimal place. [6]

8. The straight line l_1 intersects the plane π_1 at the point A, whereas the plane π_1 intersects the plane π_2 at the straight line l_2. The straight line l_1, the plane π_1 and the plane π_2 are defined as follows :

$l_1 : \frac{x+1}{2} = y - 3 = \frac{z-4}{3}$

$\pi_1 : 2x - 3y + z = 5$

$\pi_2 : r \cdot (3\underline{i} + 3\underline{j} + \underline{k}) = 8$

Find

a) the position vector of point A, [4]

b) the vector equation of the straight line l_2, [6]

c) the Cartesian equation of the plane that contains the straight line l_1 and is perpendicular to the plane π_1. [5]

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Peperiksaan Percubaan STPM 2021 Sem 1 SMK St. Paul
954/1 Mathematics T Marking Scheme

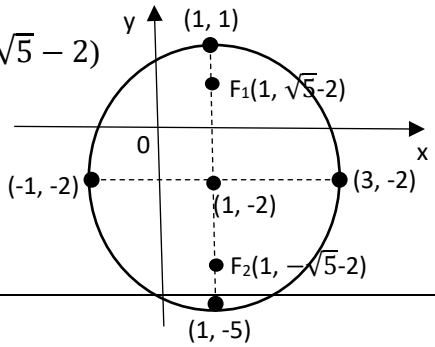
No	Scheme	Marks
1 (5)	$P(x) = x^3 + x^2 - 10x + 8$ a) $x = 2$, $P(2) = 2^3 + 2^2 + 8 = 8 + 4 - 20 + 8 = 0$ $\therefore$ $x - 2$ is a factor	M1 :find P(2) using factor theorem. A1: show = 0 and conclusion about $x - 2$ being a factor
	b) Use long division to find the quadratic factor $x^2 + 3x - 4$ $P(x) = (x - 2)(x^2 + 3x - 4)$ $= (x - 2)(x - 1)(x + 4)$	M1 attempt at quadratic factor $x^2 + 3x - 4$ Or factor theorem again for 2nd factor. A1 A1
2 (9)	a) $\left(r + \frac{1}{2}\right)^4 = r^4 + \binom{4}{1} r^3 \left(\frac{1}{2}\right) + \binom{4}{2} r^2 \left(\frac{1}{2}\right)^2 + \binom{4}{3} r^1 \left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^4$ $= r^4 + 2r^3 + \frac{3}{2}r^2 + \frac{1}{2}r + \frac{1}{16}$ b) $\left(r + \frac{1}{2}\right)^4 - \left(r - \frac{1}{2}\right)^4$ $= r^4 + 2r^3 + \frac{3}{2}r^2 + \frac{1}{2}r + \frac{1}{16} - (r^4 - 2r^3 + \frac{3}{2}r^2 - \frac{1}{2}r + \frac{1}{16})$ $= 4r^3 + r$ $\therefore 4r^3 + r \equiv \left(r + \frac{1}{2}\right)^4 - \left(r - \frac{1}{2}\right)^4$ c) $\sum_{r=1}^{n}(4r^3 + r) = \sum_{r=1}^{n} \left(r + \frac{1}{2}\right)^4 - \left(r - \frac{1}{2}\right)^4$ $= \left(\frac{3}{2}\right)^4 - \left(\frac{1}{2}\right)^4$ $+ \left(\frac{5}{2}\right)^4 - \left(\frac{3}{2}\right)^4$ $+ \left(\frac{7}{2}\right)^4 - \left(\frac{5}{2}\right)^4$ $+ \ldots$ $+ \left(n - \frac{3}{2}\right)^4 - \left(n - \frac{5}{2}\right)^4$ $+ \left(n - \frac{1}{2}\right)^4 - \left(n - \frac{3}{2}\right)^4$ $+ \left(n + \frac{1}{2}\right)^4 - \left(n - \frac{1}{2}\right)^4$ $= -\left(\frac{1}{2}\right)^4 + \left(n + \frac{1}{2}\right)^4$ $= -\frac{1}{16} + n^4 + 2n^3 + \frac{3}{2}n^2 + \frac{1}{2}n + \frac{1}{16}$ $= n^4 + 2n^3 + \frac{3}{2}n^2 + \frac{1}{2}n$	M1 use binomial expansion. A1 M1 A1: Verified A1: State the answer M1: Method of differences M1: working A1 A1

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3 (8)	$A^2B = \begin{pmatrix} 121 & 0 & 0 \\ 0 & 121 & 0 \\ 0 & 0 & 121 \end{pmatrix}$ AAB = 121 I $A\left(\frac{1}{121}AB\right) = I$ $A^{-1} = \frac{1}{121}AB$ $= \frac{1}{121}\begin{pmatrix} -1 & 2 & 1 \\ -3 & 1 & 4 \\ 0 & 1 & 2 \end{pmatrix}\begin{pmatrix} -35 & 19 & 18 \\ -27 & -13 & 45 \\ -3 & 12 & 5 \end{pmatrix}$ $= \frac{1}{121}\begin{pmatrix} -22 & -33 & 77 \\ 66 & -22 & 11 \\ -33 & 11 & 55 \end{pmatrix}$ $= \begin{pmatrix} -\frac{2}{11} & -\frac{3}{11} & \frac{7}{11} \\ \frac{6}{11} & -\frac{2}{11} & \frac{1}{11} \\ -\frac{3}{11} & \frac{1}{11} & \frac{5}{11} \end{pmatrix}$ b) $-x + 2y + z = 8$ $-3x + y + 4z = 15$ $y + 2z = 4$ $\begin{pmatrix} -1 & 2 & 1 \\ -3 & 1 & 4 \\ 0 & 1 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 8 \\ 15 \\ 4 \end{pmatrix}$ $A\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 8 \\ 15 \\ 4 \end{pmatrix}$ $A^{-1}A\begin{pmatrix} x \\ y \\ z \end{pmatrix} = A^{-1}\begin{pmatrix} 8 \\ 15 \\ 4 \end{pmatrix}$ $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = A^{-1}\begin{pmatrix} 8 \\ 15 \\ 4 \end{pmatrix}$ $= \begin{pmatrix} -\frac{2}{11} & -\frac{3}{11} & \frac{7}{11} \\ \frac{6}{11} & -\frac{2}{11} & \frac{1}{11} \\ -\frac{3}{11} & \frac{1}{11} & \frac{5}{11} \end{pmatrix}\begin{pmatrix} 8 \\ 15 \\ 4 \end{pmatrix}$ $= \begin{pmatrix} -3 \\ 2 \\ 1 \end{pmatrix}$ $\therefore$ x = −3 y = 2, z = 1	M1 M1 A1 M1 : rearrange the equations then write in matrix form. M1 M1 A1 A1 : State answer for x, y, z

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4 (8)	a) $\cos 5\theta + i\sin 5\theta$ $= (\cos\theta + i\sin\theta)^5$ $= \cos^5\theta + 5\cos^4\theta(i\sin\theta) + 10\cos^3\theta(i\sin\theta)^2$ $+ 10\cos^2\theta(i\sin\theta)^3 + 5\cos\theta(i\sin\theta)^4 + (i\sin\theta)^5$ $= \cos^5\theta + 5i\cos^4\theta\sin\theta - 10\cos^3\theta\sin^2\theta - 10i\cos^2\theta\sin^3\theta$ $+ 5\cos\theta\sin^4\theta + i\sin^5\theta$ $= \cos^5\theta - 10\cos^3\theta\sin^2\theta + 5\cos\theta\sin^4\theta$ $+ i(5\cos^4\theta\sin\theta - 10\cos^2\theta\sin^3\theta + \sin^5\theta)$ Real part: $\cos 5\theta = \cos^5\theta - 10\cos^3\theta\sin^2\theta + 5\cos\theta\sin^4\theta$ $= \cos^5\theta - 10\cos^3\theta(1 - \cos^2\theta) + 5\cos\theta(1 - \cos^2\theta)^2$ $= \cos^5\theta - 10\cos^3\theta + 10\cos^5\theta + 5\cos\theta(1 - 2\cos^2\theta + \cos^4\theta)$ $= \cos^5\theta - 10\cos^3\theta + 10\cos^5\theta + 5\cos\theta - 10\cos^3\theta + 5\cos^5\theta$ $= 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta$ $= \cos\theta(16\cos^4\theta - 20\cos^2\theta + 5)$ b) $16\cos^4\theta - 20\cos^2\theta + 5 = \frac{\cos 5\theta}{\cos\theta}$ let $x = \cos\left(\frac{1}{10}\pi\right)$ $16x^4 - 20x^2 + 5 = 16\cos^4\left(\frac{1}{10}\pi\right) - 20\cos^2\left(\frac{1}{10}\pi\right) + 5$ $= \frac{\cos 5\left(\frac{1}{10}\pi\right)}{\cos\left(\frac{1}{10}\pi\right)}$ $= \frac{\cos\left(\frac{1}{2}\pi\right)}{\cos\left(\frac{1}{10}\pi\right)} = \frac{0}{\cos\left(\frac{1}{10}\pi\right)}$ $= 0$ $\therefore x = \cos\left(\frac{1}{10}\pi\right)$ is a root of the equation	M1: use De Moivre's Theorem M1: binomial expansion or others method to expand M1: Compare real part A1: Shown M1: compare and substitute M1: simplify A1: shown A1: state answer
5 (7)	$x = 1 + a\cos\theta$, $y = b\sin\theta - 2$ a) $\cos\theta = \frac{x-1}{a}$, $\sin\theta = \frac{y+2}{b}$ $\cos^2\theta + \sin^2\theta = 1$ $\left(\frac{x-1}{a}\right)^2 + \left(\frac{y+2}{b}\right)^2 = 1$ $\therefore \frac{(x-1)^2}{a^2} + \frac{(y+2)^2}{b^2} = 1$ b) $a = 2$, $b = 3$ $\frac{(x-1)^2}{2^2} + \frac{(y+2)^2}{3^2} = 1$ Centre = (1, -2) $c^2 = 3^2 - 2^2 = 5$ $c = \pm\sqrt{5}$ Foci = $(1, -\sqrt{5} - 2)$, $(1, \sqrt{5} - 2)$  	M1 A1 B1: centre B1: foci both B1: Shape B1: label A1: all correct

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6	a)$1 + \lambda = 1 + 2\mu$ ….(1) $\lambda = 3 + \mu$ ….(2) $-1 = 6 - \mu$ ….(3) From (3), $\mu = 7$; subs into (2), $\lambda = 10$ From (1), $1 + \lambda = 1 + 2\mu$ => $1 + \lambda = 1 + 10 = 11$ $1 + 2\mu = 1 + 2(7) = 15$ $11 \neq 15$ $\therefore$ l_1 and l_2 do not interest, hence l_1 and l_2 do not meet b) $\overrightarrow{OA} = \begin{pmatrix}1\\0\\-1\end{pmatrix} + 1\begin{pmatrix}1\\1\\0\end{pmatrix} = \begin{pmatrix}2\\1\\-1\end{pmatrix}$ $\overrightarrow{OB} = \begin{pmatrix}1\\3\\6\end{pmatrix} + 2\begin{pmatrix}2\\1\\-1\end{pmatrix} = \begin{pmatrix}5\\5\\4\end{pmatrix}$ $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = \begin{pmatrix}5-2\\5-1\\4+1\end{pmatrix} = \begin{pmatrix}3\\4\\5\end{pmatrix}$ Cos $\theta = \frac{\begin{pmatrix}3\\4\\5\end{pmatrix}\cdot\begin{pmatrix}1\\1\\0\end{pmatrix}}{\sqrt{3^2+4^2+5^2}\sqrt{1+1+0}}$ $= \frac{3+4+0}{\sqrt{50}\sqrt{2}}$ $= \frac{7}{10}$	M1 : solve two equations to find μ and λ M1: subs into 3rd equation to compare and show A1: State answer M1 : Find OA or OB M1, M1: Find AB M1: Use formula A1: Answer cos θ A0: if give answer for θ only
7	a) $\cos(A+B) = \cos A \cos B - \sin A \sin B$ $\cos 2A = \cos(A+A)$ $= \cos A \cos A - \sin A \sin A$ $= \cos^2 A - \sin^2 A$ $= (1 - \sin^2 A) - \sin^2 A$ $= 1 - 2\sin^2 A$ b) $2\sin 2\theta - 3\cos 2\theta - 3\sin\theta + 3$ $= 2(2\sin\theta\cos\theta) - 3(1 - 2\sin^2\theta) - 3\sin\theta + 3$ $= 4\sin\theta\cos\theta - 3 + 6\sin^2\theta - 3\sin\theta + 3$ $= \sin\theta(4\cos\theta + 6\sin\theta - 3)$ c) $4\cos\theta + 6\sin\theta = R\sin(\theta + \alpha)$ $= R\sin\theta\cos\alpha + R\cos\theta\sin\alpha$ $= R\sin\alpha\cos\theta + R\cos\alpha\sin\theta$ $R\sin\alpha = 4$ ….(1) $R\cos\alpha = 6$ ….(2) $\frac{(1)}{(2)}$: $\tan\alpha = \frac{4}{6}$ $\alpha = 33.69° = 0.18717\pi = 0.588$ rad (3dp) $(1)^2 + (2)^2$: $R^2 = 4^2 + 6^2$ $R = \sqrt{52} = 2\sqrt{13}$ $4\cos\theta + 6\sin\theta = 2\sqrt{13}\sin(\theta + 0.588)$	M1 A1 M1: substitute M1: rearrange A1 B1: both equations M1: find α in rad M1: find R in exact value A1 : write in proper equation

	d) $2\sin2\theta - 3(\cos2\theta + \sin\theta - 1) = 0$ $\sin\theta\,(4\cos\theta + 6\sin\theta - 3) = 0$ $\sin\theta\,(2\sqrt{13}\sin(\theta + 0.588) - 3) = 0$ $\sin\theta = 0$, $\sin(\theta + 0.588) = \frac{3}{2\sqrt{13}}$ $\theta = 0$, $\theta + 0.588 = 24.584°, 155.416°$ $\theta + 0.588 = 0.4291$, 2.7125 $\theta = -0.1589, 2.1245$ $\therefore \theta = 2.124$ (3dp)	M1:Rearrange M1:Substitute M1: factorise M1: get basic angle 24.584° M1: answer for θ A1: State the correct answer correct to 3 dp within the given range
8	a) $l_1 : \frac{x+1}{2} = y - 3 = \frac{z-4}{3}$, $\pi_1 : 2x - 3y + z = 5$ $\begin{pmatrix}x\\y\\z\end{pmatrix} = \begin{pmatrix}-1\\3\\4\end{pmatrix} + \lambda\begin{pmatrix}2\\1\\3\end{pmatrix}$ $\begin{pmatrix}x\\y\\z\end{pmatrix} = \begin{pmatrix}-1+2\lambda\\3+\lambda\\4+3\lambda\end{pmatrix}$ $x = -1+2\lambda$, $y = 3 + \lambda$, $z = 4 + 3\lambda$ $2x - 3y + z = 5$ $2(-1+2\lambda) - 3(3 + \lambda) + (4 + 3\lambda) = 5$ $-2 + 4\lambda - 9 - 3\lambda + 4 + 3\lambda = 5$ $4\lambda = 12$ $\lambda = 3$ Position vector of A, $\overrightarrow{OA} = \begin{pmatrix}5\\6\\13\end{pmatrix}$ b) equation of line, $\underline{r} = \underline{a} + \lambda\underline{b}$ $\pi_2 : r \cdot (3\underline{i} + 3\underline{j} + \underline{k}) = 8$ ➔ $3x + 3y + z = 8$ $\pi_1 : 2x - 3y + z = 5$ $3x + 3y + (5 - 2x + 3y) = 8$ $x + 6y = 3$ let $y = 0$ ➔ $x = 3$, $z = -1$ $\therefore$ $(3, 0, -1)$ is a point on the line l_2 is parallel to the vector $\underline{n}_1 \times \underline{n}_2 = \begin{pmatrix}2\\-3\\1\end{pmatrix} \times \begin{pmatrix}3\\3\\1\end{pmatrix} = \begin{vmatrix}\underline{i} & \underline{j} & \underline{k}\\2 & -3 & 1\\3 & 3 & 1\end{vmatrix}$ $= [(-3)(1) - 3(1)]\underline{i} - [2(1) - 3(1)]\,\underline{j} + [2(3) - 3(-3)]\,\underline{k}$ $= -6\underline{i} + \underline{j} + 15\underline{k}$ $\therefore$ the vector equation of the line l_2 is $\underline{r} = 3\underline{i} - \underline{k} + \lambda(-6\underline{i} + \underline{j} + 15\underline{k})$	M1: Find coordinates of point A in terms of λ from l_1 M1: Substitute into plane π_1 A1: find λ A1: state position vector of A M1: Find a point of l_2 A1: Get a point M1: Find vector parallel to l_2 (the vector is perpendicular to both plane) M1: working A1: vector parallel to l_2 A1: vector equation CAO or $\underline{r} = \begin{pmatrix}3\\0\\-1\end{pmatrix} + \lambda\begin{pmatrix}-6\\1\\15\end{pmatrix}$

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