SAMPEL PERCUMA

Trial STPM 2024 Mathematics T Semester 1 Pahang

Set penuh: Kertas Percubaan STPM

Sem 1 · Mathematics T

Sampel ini: 8 muka surat. Set penuh ada:

- 3 fail PDF
- Semester 1: kertas percubaan (trial) negeri, subjek yang dipilih + skema
- Blueprint 30 hari Semester 1 (setiap subjek)
- Semester 3: percubaan 2024, 10 subjek, banyak negeri, soalan + skema dalam satu fail

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Section A [45 marks]

Answer **all** the questions in this section.

1. Show that $f(x) = -2x^3 + 1 \ , x \in R$ is a one – to – one function, sketch the graph, define its domain and range. Mention restrictions if any. [8]

2. Given $f(r) = (r-1)^2(r-2)^2$. Simplify $f(r) - f(r-1)$.
   Hence, deduce the value of $\sum_{r=1}^{n}(r-2)^3$ [5]

3. A system of linear equation is given by

$$x + 2y + z = \beta$$
$$2x + \alpha y + 4z = 3\beta$$
$$x + y + \alpha z = \beta$$

where α and β are constants.

a) write the augmented matrix for the system above and reduce it to row- echelon form. [5]
b) determine the values of α and β such that the system has a unique solution. [3]
c) from (b) deduce the determinant of the matrix.

A= $\begin{bmatrix} 1 & 2 & 1 \\ 2 & \alpha & 4 \\ 1 & 1 & \alpha \end{bmatrix}$ and justify the existence of the inverse [2]

4. The complex number z is given by $z = \frac{3}{4} + \frac{\sqrt{3}}{4}i$.
   a) State z in polar form. [3]
   b) By using de Moivre's theorem, find z^6. [2]
   c) Express $\frac{z^5}{z^*}$ in the form of $x + yi$, where z^* is the conjugate of z, where $x, y \in \Re$. [3]

5. The parametric equation of a conic is defined as

   $x = 3\sec\theta + 5$ and $y = 5tan\,\theta - 8$

   a) Show that the Cartesian equation of the conic represents a hyperbola. Hence, sketch the hyperbola. [5]
   b) State the centre, vertices, foci and asymptotes of the hyperbola. [5]

6. A line l , passes through points S (3,-1,8) and T (5,-4,7) and meets the plane

   $5x - y - 2z = 30$ at point P.

   a) Find the parametric equation of l. [3]
   b) Determine the coordinates of the point P. [3]

Section B [15 marks]
Answer one question only in this section.

7. The polynomial $p(x) = hx^4 + kx^3 + 2x - 1$, where h and k are constants, leaves a reminder of 4 when divided by $x - 1$, and a remainder of -2 when divided by $x + 1$.
   a) Determine the values of h and k. [3]
   b) Express the polynomial $p(x)$ in the form $(x^2 - 1)q(x) + r(x)$, where $q(x)$ is quadratic and $r(x)$ is linear. [4]
   c) Express $q(x)$ in a completed square form $a(x + b)^2 + c$ [2]
   i) Deduce that $q(x)$ is always positive for all real values of x. [1]
   ii) Deduce the minimum value of $q(x)$ and the corresponding value of x. [2]
   d) Determine the set of values of x for which $p(x) > 3x + 1$ [3]

8. By using the elementary row operation, find the inverse matrix of $\begin{pmatrix} 1 & 2 & 4 \\ 2 & 3 & 4 \\ 4 & 3 & 1 \end{pmatrix}$. [10]

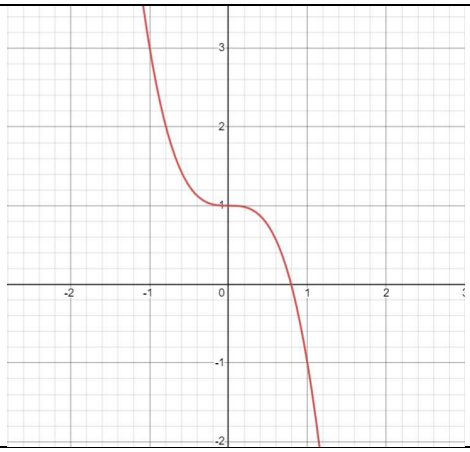
School holiday is coming. Three families went to Matta Fair to book a travel trip to China, New York and Australia. The package they have book and amount spent are shown in the following table:

Family	China	New York	Australia	Amount Paid (RM)
Albert	1	2	4	80 000
Jeffrey	2	3	4	110 000
Patrick	4	3	1	100 000

The price of package to Japan, Korea and Taiwan are RM x, RM y, and RM z respectively. Obtain a matrix equation to represent the above information. Hence, determine the price of each package trip. [5]

MARKING SCHEME MATHEMATICS T SEMESTER 1 2024 (PAHANG)

1. Show that $f(x) = -2x^3 + 1 \quad , x \in R$ is a one – to – one function, sketch the graph, define its domain and range. Mention restrictions if any. [8]

No	Suggestion Solution	marks	
1	Let $x_1, x_2 \in R$, if $x_2 > x_1$ $f(x_2) - f(x_1) = -2x_2{}^3 + 1 - (-2x_1^3 + 1)$	B1	
	$-2[x_2^3 + x_1^3] < 0 \, , \forall x \in R$	M1	
	Since $x_2 > x_1 \rightarrow x_2^3 < x_1^3$ Hence $f(x_2) < f(x_1)$	M1	
	From definition, when $x_2 > x_1$ and $f(x_2) < f(x_1)$ then f is a decreasing function Through out $x \in R$ Hence , f is one – to – one function.	A1	
	$D_f \; and R_f = \{x \mid x \in R\}$	B1	
	No restrictions	B1	
		D1 D1	Shape All correct

2. Given $f(r) = (r-1)^2(r-2)^2$. Simplify $f(r) - f(r-1)$.
Hence, deduce the value of $\sum_{r=1}^{n} (r-2)^3$. [5]

No	Suggestion Solution	marks	
2	Let $f(r) \quad = (r-1)^2(r-2)^2$ $f(r-1) = (r-2)^2(r-3)^2$		
	$f(r) - f(r-1)$ $= (r-1)^2(r-2)^2 - (r-2)^2(r-3)^2$ $= (r-2)^2[(r-1)^2 - (r-3)^2]$ $= (r-2)^2[r^2 - 2r + 1 - (r^2 - 6r + 9)]$ $= (r-2)^2[r^2 - 2r + 1 - r^2 + 6r - 9]$ $= (r-2)^2[4r - 8]$ $= (r-2)^2 4(r-2)$	M1	Substituting his $f(r)$ & $f(r-1)$ and attempt to solve
	$f(r) - f(r-1) = 4(r-2)^3$	A1	
	$4(r-2)^3 = f(r) - f(r-1)$ $\sum_{r=1}^{n} (r-2)^3 = \frac{1}{4}[f(r) - f(r-1)]$		

4. The complex number z is given by $z = \frac{3}{4} + \frac{\sqrt{3}}{4}i$.

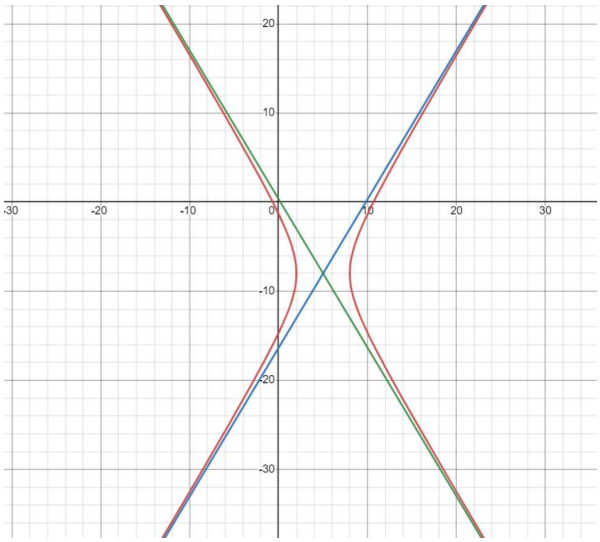
(a) State z in polar form. [3]

(b) By using de Moivre's theorem, find z^6. [2]

(c) Express $\frac{z^5}{z^*}$ in the form of $x + yi$, where z^* is the conjugate of z, where $x, y \in \Re$. [3]

No	Suggestion Solution	Mark	
a)	$\|z\| = \sqrt{\left(\frac{3}{4}\right)^2 + \left(\frac{\sqrt{3}}{4}\right)^2} = \frac{\sqrt{3}}{2}$ $\arg(z) = tan^{-1}\left(\frac{\sqrt{3}}{3}\right) = \frac{\pi}{6}$	M1 A1	Either 1 Correct Both Correct
	$z = \frac{\sqrt{3}}{2}(cos\frac{\pi}{6} + isin\frac{\pi}{6})$	A1	
b)	$z^6 = \left(\frac{\sqrt{3}}{2}(cos\frac{\pi}{6} + isin\frac{\pi}{6})\right)^6$ $= \frac{27}{64}\left(\cos\left(\frac{6\pi}{6}\right) + isin\left(\frac{6\pi}{6}\right)\right)$	M1 (his)	
	$= \frac{27}{64}(-1 + 0i) = -\frac{27}{64}$	A1	
c)	$\frac{z^5}{z^*} = \frac{z^6}{zz^*} = \frac{z^6}{\|z\|^2}$	M1	
	$\frac{\left(-\frac{27}{64}\right)}{\left(\frac{\sqrt{3}}{2}\right)^2} = -\frac{9}{16}$	M1	
	$\frac{z^5}{z^*} = -\frac{9}{16} + 0i$	A1	complex form seen

5. The parametric equation of a conic is defined as
$x = 3\sec\theta + 5$ and $y = 5tan\,\theta - 8$
Show that the Cartesian equation of the conic represents a hyperbola. Hence, sketch the hyperbola. [5]
State the centre, vertices, foci and asymptotes of the hyperbola. [5]

No	Suggestion Solution	Marks	
a)	$sec\theta = \frac{x-5}{3}$ or $tan\,\theta = \frac{y+8}{5}$	B1	
	$\tan^2\theta + 1 = \sec^2\theta$ $\sec^2\theta - \tan^2\theta = 1$	M1	(use trigonometry identities)
	$\left(\frac{x-5}{3}\right)^2 - \left(\frac{y+8}{5}\right)^2 = 1$ $\therefore$ *Shown hyperbola general form*	A1	
		D1 D1	(shape) (asymptotes)
b)	Centre:$(5,-8)$	A1	
	Vertice: $(8,-8),(2,-8)$	A1	
	$c = \sqrt{34}$ Foci: $\left(5-\sqrt{34},-8\right),\left(5+\sqrt{34},-8\right)$	A1(both correct)	
	Asymptotes: $y-(-8) = \pm\frac{5}{3}(x-5)$	M1	($\frac{5}{3}$ *seen*)
	$3y - 5x + 49 = 0$ or $3y + 5x - 1 = 0$	A1	

6. A line l , passes through points S (3,-1,8) and T (5,-4,7) and meets the plane $5x - y - 2z = 30$ at point P.

a) Find the parametric equation of l. [3]
b) Determine the coordinates of the point P. [3]

No	Suggestion Solution	Marks	
	S(3,-1,8) , T (5 ,-4 , 7) $\overrightarrow{ST} = \begin{pmatrix} 5 \\ -4 \\ 7 \end{pmatrix} - \begin{pmatrix} 3 \\ -1 \\ 8 \end{pmatrix} = \begin{pmatrix} 2 \\ -3 \\ -1 \end{pmatrix}$	B1	
	The equation of line l is $\boldsymbol{r} = \begin{pmatrix} 3 \\ -1 \\ 8 \end{pmatrix} + \gamma \begin{pmatrix} 2 \\ -3 \\ -1 \end{pmatrix}$, Where γ is a parameter. $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 + 2\gamma \\ -1 - 3\gamma \\ 8 - \gamma \end{pmatrix}$	M1	
	The parametric equations of l are $x = 3 + 2\gamma$, $y = -1 - 3\gamma$, $z = 8 - \gamma$	A1	
	$5(3 + 2\gamma) - (-1 - 3\gamma) - 2(8 - \gamma) = 30$ $15 + 10\gamma + 1 + 3\gamma - 16 + 2\gamma = 30$ $15\gamma = 30$ $\gamma = 2$	M1	
	$x = 3 + 2(2) = 7$ $y = -1 - 3(2) = -7$ $z = 8 - 2 = 6$	M1	
	The coordinates of the point P is (7, -7,6)	A1	

7. The polynomial $p(x) = hx^4 + kx^3 + 2x - 1$, where h and k are constants, leaves a reminder of 4 when divided by $x - 1$, and a remainder of -2 when divided by $x + 1$.

a) Determine the values of h and k. [3]

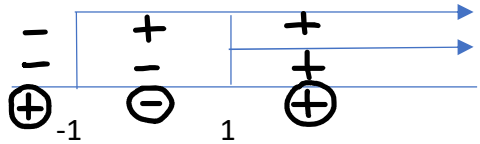
b) Express the polynomial p(x) in the form $(x^2 - 1)q(x) + r(x)$, where $q(x)$ is quadratic and $r(x)$ is linear. [4]

c) Express q(x) in a completed square form $a(x + b)^2 + c$ [2]

i) Deduce that q(x) is always positive for all real values of x. [1]

ii) Deduce the minimum value of q(x) and the corresponding value of x. [2]

d) Determine the set of values of x for which $p(x) > 3x + 1$ [3]

No	Suggestion Solution	Marks	
7 a)	$p(1) = h(1) + k(1) + 2(1) - 1 = 4$ $h + k = 3 \ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$ $p(-1) = h - k - 2 - 1 = -2$ $h - k = 1 \ldots\ldots\ldots\ldots\ldots (2)$	M1 A1	(at least 1 correct)
	$\therefore h = 2, k = 1$	A1	
b)	$p(x) = 2x^4 + x^3 + 2x - 1$ Using long division $(2x^4 + x^3 + 2x - 1) \div (x^2 - 1)$	M1	
	$= (x^2 - 1)(2x^2 + x + 2) + 3x + 1$	A1	
	Hence , $q(x) = (2x^2 + x + 2)$ and $r(x) = 3x + 1$	 B1 B1	
c)	$q(x) = (2x^2 + x + 2) = 2\left(x + \frac{1}{4}\right)^2 + \frac{15}{8}$	M1 A1	
(i)	Since $\left(x + \frac{1}{4}\right)^2 > 0$ *then* $2\left(x + \frac{1}{4}\right)^2 + \frac{15}{8} > 0$	B1	
(ii)	min value is $\frac{15}{8}$ where $x = -\frac{1}{4}$	B1B1	
d)	$p(x) = 2x^4 + x^3 + 2x - 1$ $= (x^2 - 1)(2x^2 + x + 2) + 3x + 1$ for $p(x) > 3x + 1$ $(x^2 - 1)(2x^2 + x + 2) + 3x + 1 > 3x + 1$		
	$(x^2 - 1)(2x^2 + x + 2) > 0$, *since* $2x^2 + x + 2$ *is always positive*	M1	
	Then $(x^2 - 1) > 0$; $(x + 1)(x - 1) > 0$  	M1	
	$\therefore \{x \mid x < -1 \text{ or } x > 1\}$	A1	

	$4R_3 + R_1 \rightarrow R_1'$ $-4R_3 + R_2 \rightarrow R_2'$ $A^{-1} = \frac{1}{5}\begin{pmatrix} 9 & -10 & 4 \\ -14 & 15 & -4 \\ 6 & -5 & 1 \end{pmatrix}$	$\left(\begin{array}{ccc\|ccc} 1 & 0 & 0 & \frac{9}{5} & -\frac{10}{5} & \frac{4}{5} \\ 0 & 1 & 4 & 2 & -1 & 0 \\ 0 & 0 & 1 & \frac{6}{5} & -\frac{5}{5} & \frac{1}{5} \end{array}\right)$ $\left(\begin{array}{ccc\|ccc} 1 & 0 & 0 & \frac{9}{5} & -\frac{10}{5} & \frac{4}{5} \\ 0 & 1 & 0 & -\frac{14}{5} & \frac{15}{5} & -\frac{4}{5} \\ 0 & 0 & 1 & \frac{6}{5} & -\frac{5}{5} & \frac{1}{5} \end{array}\right)$	M1 M1A1 A1	
	Must use ERO, If not use, "0" marks		10M	
	The system of linear equation: $x + 2y + 4z = 80000$ $2x + 3y + 4z = 110\,000$ $4x + 3y + z = 100\,000$			
	$\begin{pmatrix} 1 & 2 & 4 \\ 2 & 3 & 4 \\ 4 & 3 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 80000 \\ 110000 \\ 100000 \end{pmatrix}$		B1	
	$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{1}{5}\begin{pmatrix} 9 & -10 & 4 \\ -14 & 15 & -4 \\ 6 & -5 & 1 \end{pmatrix}\begin{pmatrix} 80000 \\ 110000 \\ 100000 \end{pmatrix}$		M1A1	Correct inverse
	$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 4000 \\ 26000 \\ 6000 \end{pmatrix}$		M1	
	$x = RM\ 4000, y = RM\ 26000, z = RM\ 6000$		A1	Must state x, y & z
			5M	

TAMAT SAMPEL

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