SEMESTER 1

950/1 **STPM 2023**

JABATAN PENDIDIKAN NEGERI KELANTAN

SIJIL TINGGI PERSEKOLAHAN MALAYSIA

PEPERIKSAAN PERCUBAAN
MATHEMATICS M

One and a half hours (*Satu jam setengah*)

Instructions to candidates:

DO NOT OPEN THIS QUESTIONS PAPER UNTIL YOU ARE ALLOWED TO DO SO.

Answer **all** questions in Section A.

Answer **one** question only in Section B.

Write the answers on the answer sheets provided. Begin each answer on a new page of the answer sheet. All important working should be shown.

Scientific calculators may be used. Programmable and graphic display calculators are **not** allowed.

Answers may be written in either English or Malay.

CONFIDENTIAL *

*This question paper is CONFIDENTIAL until the examination is over.

Section A [45 *marks*]
Answer **all** *questions.*

1. Functions f and g are defined by

$$f: \to 2 + e^{-x}, x \in \mathbb{R}$$

$$g: \to \ln(2x - 5), x \geq 3$$

a) Determine the range of f [*1 marks*]

b) Find the composite function gf. [*2 marks*]

c) Find the composite function fg, and state its domain and range. [*4 marks*]

2. a) Given $f(r) = (3r - 1)(3r + 1)$, find $\sum_{r=1}^{n} f(r)$ in term of n. [*3 marks*]

b) Given $g(r) = \frac{1}{3r-1}$ simplify $g(r) - g(r + 1)$.

Hence, show that $\sum_{r=1}^{n} \frac{1}{(3r-1)(3r+2)} = \frac{n}{6n+4}$ [*6marks*]

3. The variables *x*, *y* and *z* satisfy the system of linear equations

$$2x - 5y + 5z = -7$$
$$x - 2y + 3z = -1$$
$$-x + 3y = 10$$

Write a matrix equation for the system of linear equations. By using Gaussian elimination, find the variables *x*, *y* and *z*. [7 *marks*]

4. Find the coordinates of the stationary point on the curve $y = (2x - 2)e^{x}$, and determine its nature. [*5* marks]

Sketch the curve. [2 marks]

5. A region *R* is bounded by a curve *xy*= 12 and a line 3*x* + 4*y* = 30.

(a) Find the coordinates of the points of intersection of the curve *xy*= 12 and the line 3*x* + 4*y* = 30. [3 *marks*]

(b) Calculate the area of the region *R*. [3 *marks*]

6. Solve the differential equation $(1 + x)\frac{dy}{dx} = y(1 - x)$, with condition $y = 1$ when $x = 0$.

Express your answer in the form of $y = (x)$. [*9 marks*]

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Section B [15 marks]
Answer **one** *question only.*

7. Matrix A is given by A = $\begin{pmatrix} 1 & 0 & 2 \\ 3 & 1 & 6 \\ 8 & 9 & 2 \end{pmatrix}$.

(a) Find the determinant of matrix A. Deduce $\begin{vmatrix} 0 & 1 & 2 \\ 1 & 3 & 6 \\ 9 & 8 & 2 \end{vmatrix}$. [5 *marks*]

(b) By performing elementary row operations on the augmented matrix (A|I), where I is the 3×3 identity matrix, find A^{-1}. [*6 marks*]

(c) Hence, solve the simultaneous equation

$3x + y + 6z = 2$
$8x + 9y + 2z = -1$
$x + 2z = 1$ [*4 marks*]

8. The curve C has equation $y = \frac{x^3(10x\sqrt{x}-256)}{\sqrt{x}}$ $x \in R, x > 0$

a) Determine expression for $\frac{dy}{dx}$. [*2 marks*]

b) Show that the y coordinate of the stationary point of C is $-m\sqrt[3]{4}$, where m is a positive integer. [*8 marks*]

c) Evaluate $\frac{d^2y}{dx^2}$ at the stationary point C. Give your answer in term of $\sqrt[3]{a}$. Hence, determine it nature. [*5 marks*]

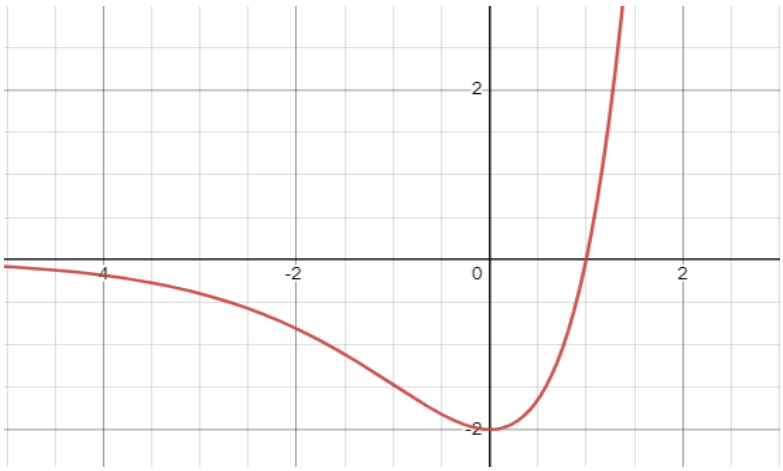
End of The Question Paper

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Answer :

1.	a) The range of f is $\{y: y \in R, y > 2\}$	B1	
	a) $g[f(x)] = g[2 + e^{-x}]$ $= \ln(2(2 + e^{-x}) - 5)$ $= \ln(4 + 2e^{-x} - 5)$ $= \ln(2e^{-x} - 1)$	M1 A1	
	b) $f[g(x)] = 2 + e^{-\ln(2x-5)}$ $= 2 + \frac{1}{2x-5}$ The composite function is $f \circ g: x \to 2 + \frac{1}{2x-5}$ Domain $f \circ g$ is $\{x: x \in R, x \geq 3\}$ Range $f \circ g$ is $\{y: y \in R, 2 < y \leq 3\}$	M1 A1 B1 B1	**7**
2.	a) $f(r) = (3r-1)(3r+1) = 9r^2 - 1$ $\sum_{r=1}^{n} f(r) = \sum_{r=1}^{n} (9r^2 - 1)$ $= 9\sum_{r-1}^{n} r^2 - n$ $= 9\left[\frac{1}{6}n(n+1)(2n+1)\right] - n$ $= \frac{1}{2}n(6n^2 + 9n + 1)$	 M1 M1 A1	
	b) $g(r) - g(r+1) = \frac{1}{3r-1} - \frac{1}{3r+2}$ $= \frac{3r + 2 - (3r-1)}{(3r-1)(3r+2)}$ $= \frac{3}{(3r-1)(3r+2)}$ $\sum_{r=1}^{n} \frac{1}{(3r-1)(3r+2)} = \frac{1}{3}\sum_{r=1}^{n} [g(r) - g(r+1)]$ $= \frac{1}{3}[g(1) - g(n+1)]$ $= \frac{1}{3}\left[\frac{1}{2} - \frac{1}{3n+2}\right]$ $= \frac{1}{3}\left[\frac{3n}{2(3n+2)}\right]$ $= \frac{n}{6n+4}$	M1 A1 B1 M1 M1 A1	**9**
3.	$\begin{pmatrix} 2 & -5 & 5 \\ 1 & -2 & 3 \\ -1 & 3 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -7 \\ -1 \\ 10 \end{pmatrix}$	B1 B1	

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	$\left(\begin{array}{ccc\|c} 2 & -5 & 5 & -7 \\ 1 & -2 & 3 & -1 \\ -1 & 3 & 0 & 10 \end{array}\right)$ $\xrightarrow{R_1 \leftrightarrow R_2} \left(\begin{array}{ccc\|c} 1 & -2 & 3 & -1 \\ 2 & -5 & 5 & -7 \\ -1 & 3 & 0 & 10 \end{array}\right)$ $\xrightarrow{\begin{array}{l} R_2 = -2R_1 + R_2 \\ R_3 = R_1 + R_3 \end{array}} \left(\begin{array}{ccc\|c} 1 & -2 & 3 & -1 \\ 0 & -1 & -1 & -5 \\ 0 & 1 & 3 & 9 \end{array}\right)$ $\xrightarrow{\begin{array}{l} R_2 = -R_2 \\ R_3 = R_2 + R_3 \end{array}} \left(\begin{array}{ccc\|c} 1 & -2 & 3 & -1 \\ 0 & 1 & 1 & 5 \\ 0 & 0 & 2 & 4 \end{array}\right)$ $2z = 4 \Rightarrow z = 2$ $y + z = 5 \Rightarrow y + 2 = 5 \Rightarrow y = 3$ $x - 2y + 3z = -1 \Rightarrow x - 2(3) + 3(2) = -1 \Rightarrow x = -1$ $x = -1,\ y = 3,\ z = 2$	M1 for attempt M1 M1 M1 A1	**7**
4.	$\frac{dy}{dx} = (2x - 2)e^x + 2e^x = 0$ $2xe^x = 0$ $x = 0, y = -2$ Coordinates on the stationary point are $(0, -2)$ $\frac{d^2y}{dx^2} = 2xe^x + 2e^x = (2x + 2)e^x$ At $(0, -2)$, $\frac{d^2y}{dx^2} = 2 > 0$ Therefore the point $(0, -2)$ is a minimum point.  	M1 A1 A1 M1 A1 D2	**7**

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6.	$\int \frac{1}{y}dy = \int \frac{1-x}{1+x}dx$	1	
	let u = 1 + x, $\frac{du}{dx} = 1$; 1 – x = 2 - u	1	
	$\int \frac{1-x}{1+x}dx = \int \frac{2-u}{u}du$	1	
	$= \int \frac{2}{u} - 1\, du$ = 2 ln u – u = 2 ln (1 + x) – (1 + x)	1	
	$\int \frac{1}{y}dy = \int \frac{1-x}{1+x}dx$ ln y = 2 ln (1 + x) – (1 + x) + c	1	
	y = 1 when x = 0, c = 1	1	
	ln y = 2 ln (1 + x) – (1 + x) + 1 ln y = 2 ln (1 + x) – x	1	
	$ln\frac{y}{(1+x)^2} = -x$		
	$\frac{y}{(1+x)^2} = e^{-x}$	1	
	$y = (1+x)^2e^{-x}$	1	**9**
7.	(a) $\|A\| = 1\begin{vmatrix}1 & 6\\ 9 & 2\end{vmatrix} - 0\begin{vmatrix}3 & 6\\ 8 & 2\end{vmatrix} + 2\begin{vmatrix}3 & 1\\ 8 & 9\end{vmatrix}$	1	
	= 1(2 – 54) – 0(6 – 48) + 2(27 – 8)	1	
	= -14	1	
	$\begin{vmatrix}0 & 1 & 2\\ 1 & 3 & 6\\ 9 & 8 & 2\end{vmatrix} = -(-14)$	1	Opposite sign with his det.
	= 14	1	
	(b) $(A\|I) = \left(\begin{array}{ccc\|ccc} 1 & 0 & 2 & 1 & 0 & 0\\ 3 & 1 & 6 & 0 & 1 & 0\\ 8 & 9 & 2 & 0 & 0 & 1\end{array}\right)$	1	CAO
	$\xrightarrow[R_3-8R_1\to R_3]{R_2-3R_1\to R_2} \left(\begin{array}{ccc\|ccc} 1 & 0 & 2 & 1 & 0 & 0\\ 0 & 1 & 0 & -3 & 1 & 0\\ 0 & 9 & -14 & -8 & 0 & 1\end{array}\right)$	1	
	$\xrightarrow{R_3-9R_2\to R_3} \left(\begin{array}{ccc\|ccc} 1 & 0 & 2 & 1 & 0 & 0\\ 0 & 1 & 0 & -3 & 1 & 0\\ 0 & 0 & -14 & 19 & -9 & 1\end{array}\right)$	1	Obtain 0 at (2,1) and (3,1) Obtain 0 at (1,2)

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	$\xrightarrow{-\frac{1}{14}R_3 \to R_3} \left(\begin{array}{ccc\|ccc} 1 & 0 & 2 & 1 & 0 & 0 \\ 0 & 1 & 0 & -3 & 1 & 0 \\ 0 & 0 & 1 & -\frac{19}{14} & \frac{9}{14} & -\frac{1}{14} \end{array}\right)$	1	and (3,2)
	$\xrightarrow{R_1 - 2R_3 \to R_1} \left(\begin{array}{ccc\|ccc} 1 & 0 & 0 & \frac{26}{7} & -\frac{9}{7} & \frac{1}{7} \\ 0 & 1 & 0 & -3 & 1 & 0 \\ 0 & 0 & 1 & -\frac{19}{14} & \frac{9}{14} & -\frac{1}{14} \end{array}\right)$	1	Obtain identity matrix at left side All correct
	$A^{-1} = \begin{pmatrix} \frac{26}{7} & -\frac{9}{7} & \frac{1}{7} \\ -3 & 1 & 0 \\ -\frac{19}{14} & \frac{9}{14} & -\frac{1}{14} \end{pmatrix}$	1	
	(c) $\begin{pmatrix} 1 & 0 & 2 \\ 3 & 1 & 6 \\ 8 & 9 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$	1	Multiply with his inverse of A
	$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \frac{26}{7} & -\frac{9}{7} & \frac{1}{7} \\ -3 & 1 & 0 \\ -\frac{19}{14} & \frac{9}{14} & -\frac{1}{14} \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$	1	
	$= \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$	1	
	$\therefore x = 1$, y = -1, z = 0		**15**
8.	a) $y = \frac{x^3(10x\sqrt{x} - 256)}{\sqrt{x}}$		
	$= 10x^4 - 256^{\frac{5}{2}}$	M1	
	$\frac{dy}{dx} = 40x^3 - 640x^{\frac{3}{2}}$	A1	
	b) $\frac{dy}{dx} = 0$		
	$40x^3 - 640x^{\frac{3}{2}} = 0$	B1	
	$x^3 - 16x^{\frac{3}{2}} = 0$		
	$x^{\frac{3}{2}}\left(x^{\frac{3}{2}} - 16\right) = 0$	M1	
	$x = 0$ @ $x = 16^{\frac{2}{3}}$	A1	
	$x = 16^{\frac{2}{3}}$		
	$x = 2^{\frac{8}{3}}, x > 0$	A1	
	$y = 10x^4 - 256x^{\frac{5}{2}}$		
	$y = 10\left(2^{\frac{8}{3}}\right)^4 - 256\left(2^{\frac{8}{3}}\right)^{\frac{5}{2}}$	B1	

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Suka sampel ini?

Kertas Percubaan STPM

Sem 1 · Mathematics M

- Semester 1: kertas percubaan (trial) negeri, subjek yang dipilih + skema
- Blueprint 30 hari Semester 1 (setiap subjek)
- Semester 3: percubaan 2024, 10 subjek, banyak negeri, soalan + skema dalam satu fail

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