Section A [45 marks]

*Answer **all** questions in this section.*

1 The functions f and g are defined by

$$f: x \to \frac{1}{2}\ln x, \quad x > 0;$$
$$g: x \to \sqrt{x}, \quad x \geq 0,$$

(a) Sketch the graph of f, and explain why the inverse function f^{-1} exists. [3 *marks*]
(b) Find f^{-1} and state its domain. [3 *marks*]
(c) Find the composite function $g \circ f^{-1}$ and state its range. [3 *marks*]

2 Express $\frac{2}{4r^2-1}$ in partial fractions. [4 *marks*]

Hence, express $S_n = \sum_{r=1}^{n} \frac{2}{4r^2-1}$ in its simplest form and find $\lim_{n\to\infty} S_n$. [4 *marks*]

3 (a) Given $A = \begin{pmatrix} t-2 & 4 & 3 \\ 1 & t+1 & -2 \\ 0 & 0 & t-4 \end{pmatrix}$, find the values of t if A is a singular matrix. [4 *marks*]

(b) Matrices A and B are defined as:

$$A = \begin{pmatrix} x & 2 & 3 \\ 0 & y & 1 \\ z & 1 & 3 \end{pmatrix}, \quad B = \begin{pmatrix} 11 & -3 & -10 \\ x & 0 & -1 \\ -4 & z & y \end{pmatrix}$$

Given that AB is commutative, find the values of *x*, *y* and *z*. [5 *marks*]

4 Given that $y = \frac{e^{-x}}{(1+x^2)}$, show that $(1+x^2)\frac{dy}{dx} + y(1+x)^2 = 0$. [5 *marks*]

5 The curve $y = x^4 + ax^2 + bx + c$ intersects *y*-axis at the point (0, 10) and has stationary points at $x = 1$ and $x = -2$. Find the values of *a*, *b* and *c*.
Find the coordinates of the stationary points and determine their nature. [7 *marks*]

6 Use a suitable substitution, show that $\int_0^1 \frac{e^x}{e^x+1} dx = ln\frac{e+1}{2}$.
Hence, show that $\int_0^1 \frac{1}{1+e^x} dx = ln\frac{2e}{e+1}$. [7 *marks*]

Section B [15 marks]

*Answer any **one** question in this section.*

1 Find the range of each of the following functions:

a) $f : x \rightarrow x^2 - x, x \in \Re$. [2 *marks*]

b) $g : x \rightarrow \frac{1}{x^2 - x}$, $x \in \Re, x \neq 0, x \neq 1$. [2 *marks*]

On separate diagrams, sketch the graphs of f and g. [5 *marks*]

Find $\lim_{x->1^-} g(x)$ and $\lim_{x->1^+} g(x)$. Hence, show that g is not continuous at $x = 1$. [3 *marks*]

Determine if g is continuous or not at $x = 0$. [3 *marks*]

2 Expand $(1 - x)^{-p}(1 - x^2)^{-q}$ in ascending powers of x, up to and including the term in x^3. [6 *marks*]

(a) If the first three terms in the above expansion are $1 - 2x + \frac{10}{9}x^2$, find the values of p and q. [3 *marks*]

(b) If $q = -\frac{1}{8}$ and the value of p is such that the coefficient of x^2 has the smallest value, show that the coefficient of x^3 in the above expansion is zero. [6 *marks*]

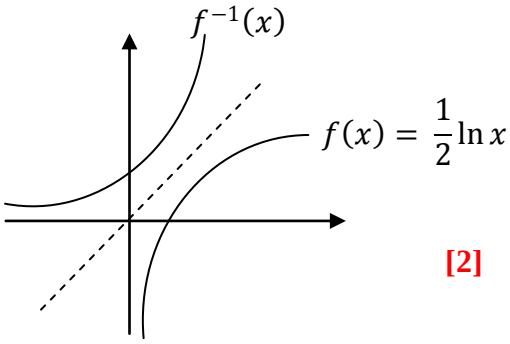
END OF QUESTION PAPER

SOLUTIONS

Section A:

1(a) Given, $f: x \to \frac{1}{2}\ln x, \quad x > 0;$

$g: x \to \sqrt{x}, \quad x \geq 0,$



[2]

From the graph of f, it is seen that f is one-to-one and f^{-1} also is one-to-one, hence the inverse function f^{-1} exists. **[1]**

1(b) $f(x) = \frac{1}{2}\ln x$

$let, \quad y = \frac{1}{2}\ln x$

$2y = \ln x$

$e^{2y} = x$

$f^{-1}(x) = e^{2x}, x \in R$ **[2, 1]**

1(c) $g \circ f^{-1} = gf^{-1}(x)$

$= g(e^{2x})$

$= \sqrt{e^{2x}}$

$= e^{x}$ **[2]**

Range: R^{+} **[1]**

2 $Let, \quad \frac{2}{4r^2-1} \equiv \frac{A}{2r-1} + \frac{B}{2r+1}$ **[1]**

$2 \equiv A(2r+1) + B(2r-1)$

$A = 1, \quad B = -1$ **[1, 1]**

$\frac{2}{4r^2-1} = \frac{1}{2r-1} - \frac{1}{2r+1}$ **[1]**

$Let, f(r) = \dfrac{1}{2r+1}$

then, $f(r-1) = \dfrac{1}{2(r-1)+1}$

$= \dfrac{1}{2r-1}$ **[1]**

$$S_n = \sum_{r=1}^{n} \frac{2}{4r^2-1}$$

$= \sum_{r=1}^{n} \left(\frac{1}{2r-1} - \frac{1}{2r+1}\right)$

$= -\left(\frac{1}{2n+1} - \frac{1}{0+1}\right)$ **[1]**

$= 1 - \frac{1}{2n+1}$ **[1]**

$\lim_{n\to\infty} S_n = 1$ **[1]**

3(a) $A = \begin{pmatrix} t-2 & 4 & 3 \\ 1 & t+1 & -2 \\ 0 & 0 & t-4 \end{pmatrix}$

$\det A = \begin{vmatrix} t-2 & 4 & 3 \\ 1 & t+1 & -2 \\ 0 & 0 & t-4 \end{vmatrix} = 0$ (A is a singular matrix) **[1]**

$(t-2)\begin{vmatrix} t+1 & -2 \\ 0 & t-4 \end{vmatrix} - 4\begin{vmatrix} 1 & -2 \\ 0 & t-4 \end{vmatrix} + 3\begin{vmatrix} 1 & t+1 \\ 0 & 0 \end{vmatrix} = 0$ **[1]**

$(t-2)(t-4)(t+1) - 4(t-4) + 3(0) = 0$

$(t-4)(t^2 - t - 2 - 4) = 0$

$(t-4)(t+2)(t-3) = 0$ **[1]**

$\therefore t = -2, 3, 4$ **[1]**

3(b) $A = \begin{pmatrix} x & 2 & 3 \\ 0 & y & 1 \\ z & 1 & 3 \end{pmatrix}$, $B = \begin{pmatrix} 11 & -3 & -10 \\ x & 0 & -1 \\ -4 & z & y \end{pmatrix}$

$AB = BA$ (AB is commutative)

$$\begin{pmatrix} x & 2 & 3 \\ 0 & y & 1 \\ z & 1 & 3 \end{pmatrix}\begin{pmatrix} 11 & -3 & -10 \\ x & 0 & -1 \\ -4 & z & y \end{pmatrix} = \begin{pmatrix} 11 & -3 & -10 \\ x & 0 & -1 \\ -4 & z & y \end{pmatrix}\begin{pmatrix} x & 2 & 3 \\ 0 & y & 1 \\ z & 1 & 3 \end{pmatrix}$$ **[1]**

$0 - y + y = 3x + 0 - 3$ **[M1]**

$3x = 3$

$x = 1$ **[1]**

$11x + 2x - 12 = 11x + 0 - 10z$

$2x + 10z - 12 = 0$

When x =1

$2(1) + 10z = 12$

$z = 1$ **[1]**

$0 + xy - 4 = x^2 + 0 - z$
When $x = 1$ and $z = 1$
$(1)y - 4 = 1^2 - 1$
$y = 4$ **[1]**

4 $y = \frac{e^{-x}}{(1+x^2)}$
$y(1 + x^2) = e^{-x}$ …..(1)

Differentiate (1) implicitly,
$2xy + (1 + x^2)\frac{dy}{dx} = -e^{-x}$ ……(2) **[2]**

Substitute (1) into (2)
$2xy + (1 + x^2)\frac{dy}{dx} = -y(1 + x^2)$ **[1]**
$(1 + x^2)\frac{dy}{dx} + 2xy + y(1 + x^2) = 0$
$(1 + x^2)\frac{dy}{dx} + y(x^2 + 2x + 1) = 0$ **[1]**

$(1 + x^2)\frac{dy}{dx} + y(1 + x)^2 = 0$ (Shown) **[1]**

5 $y = x^4 + ax^2 + bx + c$
When $x = 0, y = 10$,
$c = 10$ **[1]**
$\frac{dy}{dx} = 4x^3 + 2ax + b$
When $x = 1$, $4 + 2a + b = 0$ ……(1)
When $x = -2$, $-32 - 4a + b = 0$ ……(2)
(1) - (2) $a = -6$ **[1]**
$b = 8$ **[1]**

∴Stationary points are (1, 13) and (- 2, - 14) **[1]**

$\frac{d^2y}{dx^2} = 12x^2 - 12$
When $x = 1$, $\frac{d^2y}{dx^2} = 0$ **[M1]**
∴ (1, 13) is a point of inflexion. **[1]**

When $x = -2$, $\frac{d^2y}{dx^2} = 36 > 0$
∴ (- 2, - 14) is a minimum point. **[1]**

950/1

MATHEMATICS M
PAPER 1
One and a half hours

SMK MUHIBBAH, SANDAKAN

Section A [45 marks]

Answer **all** questions in this section.

1. Solve the equations

   (a) $3e^{-2x} + 2 - e^{2x} = 0$. [5 marks]

   (b) $\ln(x + 2) = 2 + lnx$ [3 marks]

   Give your answers correct to 3 decimal places.

2. Express $\frac{2}{4r^2 - 1}$ in partial fractions. [2 marks]

   Hence, express $S_n = \sum_{r=1}^{n} \frac{4}{4r^2 - 1}$ in its simplest form and find S_∞. [5 marks]

3. Solve the following system of linear equations using Gaussian elimination.

   $x - 2y + z = 2$

   $2x - 3y + 4z = 5$

   $-x + 3y + 2z = 1$ [7 marks]

4. Given $y(1 + x^2) = x$, show that $x^2 \frac{dy}{dx} = (1 - x^2)y^2$ [4 marks]

   Hence, find the value of $\frac{dy}{dx}$ when $x=2$ [3 marks]

5. Evaluate the definite integrals

   (a) $\int_0^7 x(x+1)^{1/3}\, dx$ [4 marks]

   (b) $\int_0^1 x^2 e^X\, dx$ [4 marks]

6. By using the substitution $y = ux^2$, show that the differential equation $x^2\frac{dy}{dx} - 2xy + 3 = 0$ may be reduced to $\frac{du}{dx} = -\frac{3}{x^4}$. [3 marks]

Hence, find the particular solution for y when x = ½ and y = 1. [5 marks]

Section B [15 marks]

Answer any **one** question in this section.

7. Given $p(x) = 3x^2 + 5x - 2$, $x \in R$.

(a) Express $p(x)$ in the form of $a(x + b)^2 + c$, with a, b and c as constants.

Hence, state the minimum value of $p(x)$. [3 marks]

(b) If $p(x)$ is factor for polynomial $q(x) = 3x^4 + 8x^3 + kx^2 + \frac{k}{2}x - 2$, find k.

By using this value of k, find the real zeroes of $q(x)$. [5 marks]

(c) Find the set values of x where

(i) $x\,[p(x)] > 0$ [3 marks]

(ii) $x[q(x)] > 0$ [4 marks]

8. A function is defined by $f(x) = \dfrac{x}{(x+2)(x-1)}$.

(a) Show that $f'(x) < 0$ for all values of x in the domain of f. [5 marks]

(b) Sketch the graph of $y = f(x)$.

State whether f is a one-one function, give reason to your answer. [5 marks]

(c) Sketch the graph of $y = |f(x)|$.

State the number of roots of the equations $|f(x)| = 2(x-1)$ [5 marks]

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Suka sampel ini?

Kertas Percubaan STPM

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- Semester 1: kertas percubaan (trial) negeri, subjek yang dipilih + skema
- Blueprint 30 hari Semester 1 (setiap subjek)
- Semester 3: percubaan 2024, 10 subjek, banyak negeri, soalan + skema dalam satu fail

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