Section A [45 *marks*]

Answer **all** *questions in this section*.

1. Determine the set of values of *x* satisfying the inequality

$$\frac{x}{x+1} < \frac{2}{x+2}$$

[5 *marks*]

2. (a) Show that, for $r > 0$,

$$r - 3 + \frac{1}{r+1} - \frac{1}{r+2} = \frac{r^3 - 7r - 5}{(r+1)(r+2)}$$

[2 *marks*]

(b) Hence, prove by using the method of differences, that

$$\sum_{r=1}^{n} \frac{r^3 - 7r - 5}{(r+1)(r+2)} = \frac{n(n^2 + an + b)}{2(n+2)}$$

where *a* and *b* are constants to be found. [5 *marks*]

3. Consider the following system of equations

$$\begin{aligned} 2x + y + 6z &= 0 \\ 4x + 3y + 14z &= 4 \\ 2x - 2y + (\alpha - 2)z &= \beta - 12 \end{aligned}$$

where α and β are constants.

(a) Write the augmented matrix for the system above and reduce it to row-echelon form. [5 *marks*]

(b) Determine the values of α and β such that the system has
(i) no solution,
(ii) a unique solution. [3 *marks*]

4. (a) Find the roots of the equation $z^4 = 8(\sqrt{3} + i)$ in Cartesian form. [6 *marks*]
(b) Show these roots on an Argand diagram. [2 *marks*]

5. The equation of the tangent to the ellipse

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

at any point $(a\cos\theta, b\sin\theta)$ on the ellipse is given by

$$\frac{x}{a}\cos\theta+\frac{y}{b}\sin\theta=1$$

(a) Tangents are drawn to the ellipse at the points $(a\cos\alpha, b\sin\alpha)$ and $(a\cos\beta, b\sin\beta)$. The tangents are perpendicular. Show that

$$\tan\alpha\tan\beta=-\frac{b^2}{a^2}$$

[4 *marks*]

(b) A line joins $P(a\cos\theta, b\sin\theta)$ to the fixed point $S(a, 0)$ on this ellipse. As θ varies, find the cartesian equation of the locus of the midpoint of the chord *PS*. [5 *marks*]

6. The lines l_1 and l_2 have equations $r=\begin{pmatrix}5\\-3\\p\end{pmatrix}+s\begin{pmatrix}0\\1\\-3\end{pmatrix}$ and $r=\begin{pmatrix}8\\5\\-2\end{pmatrix}+t\begin{pmatrix}3\\4\\-5\end{pmatrix}$ respectively, where *s* and *t* are parameters and *p* is a constant.

The lines l_1 and l_2 intersect at the point *A*.

(a) Find the coordinates of *A*. [2 *marks*]

(b) Find the value of the constant *p*. [3 *marks*]

(c) Find the acute angle between l_1 and l_2, giving your answer in degrees to 2 decimal places. [3 *marks*]

Section B [15 *marks*]

Answer any **one** *question in this section.*

7. (a) Express $2sin\theta - 1.5cos\theta$ in the form $Rsin(\theta - \alpha)$, where $R > 0$ and $0 < \theta < \frac{\pi}{2}$.

Give the value of α to 4 decimal places. [3 *marks*]

(b) (i) Find the maximum value of $2sin\theta - 1.5cos\theta$.

(ii) Find the value of θ, for $0 \leq \theta < \pi$, at which this maximum occurs. [3 *marks*]

Majid models the height of sea water, *H* meters, on a particular day by the equation

$$H = 6 + 2sin\left(\frac{4\pi t}{25}\right) - 1.5cos\left(\frac{4\pi t}{25}\right), \qquad 0 \leq t < 12,$$

where *t* hours is the number of hours after midday.

(c) Calculate the maximum value of *H* predicted by this model and the value of *t*, to 2 decimal places, when this maximum occurs. [3 *marks*]

(d) Calculate, to the nearest minute, the times when the height of sea water is predicted by this model to be 7 meters. [6 *marks*]

8. A tetrahedron has vertices *A* (1, 2, 1), *B* (0, 1, 0), *C* (2, 1, 3) and *D* (10, 5, 5).
Find

a) a Cartesian equation of the plane *ABC*. [4 *marks*]

b) the volume of the tetrahedron *ABCD*. [4 *marks*]

The plane ***p*** has equation $2x - 3y + 3 = 0$.

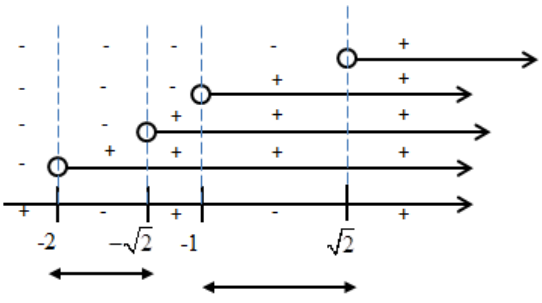
The point *E* lies on the line *AC* and the point *F* lies on the line *AD*.

Given that ***p*** contains the point *B*, the point *E* and the point *F*,

c) find the value of *k* such that $\overrightarrow{AE} = k\overrightarrow{AC}$. [4 *marks*]

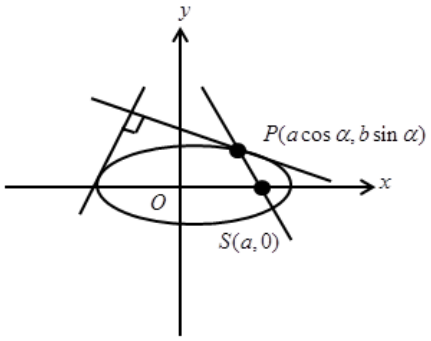
d) Given that $\overrightarrow{AF} = \frac{1}{9}\overrightarrow{AD}$, show that the volume of the tetrahedron *ABCD* is 45 times the volume of the tetrahedron *ABEF*. [3 *marks*]

Marking Scheme : Pra Pentaksiran STPM 2021 Sem 1 Daerah Muar

No		Sub mark	Total mark
1	Determine the set of values of x satisfying the inequality $\frac{x}{x+1} < \frac{2}{x+2}$ [5 *marks*] Solution : $\frac{x}{x+1} < \frac{2}{x+2}$ $\frac{x}{x+1} - \frac{2}{x+2} < 0$ $\frac{x(x+2)-2(x+1)}{(x+1)(x+2)} < 0$ $\frac{x^2-2}{(x+1)(x+2)} < 0$ For number line : $x^2-2>0, \quad x+1>0, \quad x+2>0$ $(x+\sqrt{2})(x-\sqrt{2})>0, \qquad x>-1, \qquad x>-2$ $x>-\sqrt{2},\ x>\sqrt{2}$  Solution = $\{x : -2 < x < -\sqrt{2} \quad or \quad -1 < x < \sqrt{2}\}$	M1 A1 M1 A1A1	5

No		Sub mark	Total mark
2	(a) Show that, for $r > 0$, $$r-3+\frac{1}{r+1}-\frac{1}{r+2}=\frac{r^3-7r-5}{(r+1)(r+2)}$$ [2 *marks*] (b) Hence, prove by using the method of differences, that $$\sum_{i=1}^{n}\frac{r^3-7r-5}{(r+1)(r+2)}=\frac{n(n^2+an+b)}{2(n+2)}$$ where a and b are constants to be found. [5 *marks*] Solution : (a) $r-3+\frac{1}{r+1}-\frac{1}{r+2}=\frac{(r-3)(r+1)(r+2)+(r+2)-(r+1)}{(r+1)(r+2)}$ $=\frac{(r-3)(r^2+3r+2))+(r+2)-(r+1)}{(r+1)(r+2)}$ $=\frac{r^3+3r^2+2r-3r^2-9r-5}{(r+1)(r+2)}$ $=\frac{r^3-7r-5}{(r+1)(r+2)}$ (b) $\sum_{r=1}^{n}\frac{r^3-7r-5}{(r+1)(r+2)}=\sum_{r=1}^{n}\left[r-3+\frac{1}{r+1}-\frac{1}{r+2}\right]$ $=\sum_{r=1}^{n}\left[r-3+\frac{1}{r+1}-\frac{1}{r+2}\right]$ $=\sum_{r=1}^{n}[r-3]-\sum_{r=1}^{n}\left[\frac{1}{r+2}-\frac{1}{r+1}\right]$ $=\sum_{r=1}^{n}r-3n-\left[\frac{1}{n+2}-\frac{1}{2}\right]$ $=\frac{1}{2}n(n+1)-3n-\frac{1}{n+2}+\frac{1}{2}$ $=\frac{n(n^2-3n-9)}{2(n+2)}$ $\therefore\ a=-3,\ b=-9$	M1 A1 B1 M1 M1 M1 A1	7

No		Sub mark	Total mark
3	Consider the following system of equations $$2x+y+6z=0$$ $$4x+3y+14z=4$$ $$2x-2y+(\alpha-2)z=\beta-12$$ where α and β are constants. (a) Write the augmented matrix for the system above and reduce it to row-echelon form. [5 *marks*] (b) Determine the values of α and β such that the system has (i) no solution, (ii) a unique solution. [3 *marks*] Solution : (a) $\left(\begin{array}{ccc\|c} 2 & 1 & 6 & 0 \\ 4 & 3 & 14 & 4 \\ 2 & -2 & \alpha-2 & \beta-12 \end{array}\right)$ $\xrightarrow{\substack{R_2=-2R_1+R_2 \\ R_3=-R_1+R_3}} \left(\begin{array}{ccc\|c} 2 & 1 & 6 & 0 \\ 0 & 1 & 2 & 4 \\ 0 & -3 & \alpha-8 & \beta-2 \end{array}\right)$ $\xrightarrow{R_3=3R_2+R_3} \left(\begin{array}{ccc\|c} 2 & 1 & 6 & 0 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & \alpha-2 & \beta \end{array}\right)$ (b) (i) $\alpha-2=0 \Rightarrow \alpha=2, \quad \beta\neq 0$ (ii) $\alpha\neq 2$	B1 M1 M1 M1 A1(all correct) A1A1 A1	8

No		Sub mark	Total mark
5	The equation of the tangent to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ at any point $(a\cos\theta,\ b\sin\theta)$ on the ellipse is given by $\frac{x}{a}\cos\theta+\frac{y}{b}\sin\theta=1$ (a) Tangents are drawn to the ellipse at the points $(a\cos\alpha,\ b\sin\alpha)$ and $(a\cos\beta,\ b\sin\beta)$. The tangents are perpendicular. Show that $\tan\alpha\tan\beta=-\frac{b^2}{a^2}$ [4 *marks*] (b) A line joins P $(a\cos\theta,\ b\sin\theta)$ to the fixed point S $(a,\ 0)$ on this ellipse. As θ varies, find the cartesian equation of the locus of the midpoint of the chord PS. [5 *marks*] Solution : (a) tangent at$(a\cos\alpha, b\sin\alpha)$: $\frac{x}{a}\cos\alpha+\frac{y}{b}\sin\alpha=1$ $\frac{y}{b}\sin\alpha=-\frac{x}{a}\cos\alpha+1$ $y=\frac{b}{\sin\alpha}\left(-\frac{x}{a}\cos\alpha+1\right)$ $y=-\frac{b}{a}\cot\alpha+\frac{b}{\sin\alpha}$  Gradient of tangent at$(a\cos\alpha, b\sin\alpha)$: : $m_1=-\frac{b}{a}\cot\alpha$ Gradient of tangent at$(a\cos\beta, b\sin\beta)$: : $m_2=-\frac{b}{a}\cot\beta$	M1 A1	

No		Sub mark	Total mark
6	The lines l_1 and l_2 have equations $r = \begin{pmatrix} 5 \\ -3 \\ p \end{pmatrix} + s\begin{pmatrix} 0 \\ 1 \\ -3 \end{pmatrix}$ and $r = \begin{pmatrix} 8 \\ 5 \\ -2 \end{pmatrix} + t\begin{pmatrix} 3 \\ 4 \\ -5 \end{pmatrix}$ respectively, where *s* and *t* are parameters and *p* is a constant. The lines l_1 and l_2 intersect at the point *A*. (a) Find the coordinates of *A*. [3 *marks*] (b) Find the value of the constant *p*. [2 *marks*] (c) Find the acute angle between l_1 and l_2, giving your answer in degrees to 2 decimal places. [3 *marks*] Solution : $l_1 : r = \begin{pmatrix} 5 \\ -3 \\ p \end{pmatrix} + s\begin{pmatrix} 0 \\ 1 \\ -3 \end{pmatrix}$ $l_2 : r = \begin{pmatrix} 8 \\ 5 \\ -2 \end{pmatrix} + t\begin{pmatrix} 3 \\ 4 \\ -5 \end{pmatrix}$ (a) $\begin{pmatrix} 5 \\ -3 \\ p \end{pmatrix} + s\begin{pmatrix} 0 \\ 1 \\ -3 \end{pmatrix} = \begin{pmatrix} 8 \\ 5 \\ -2 \end{pmatrix} + t\begin{pmatrix} 3 \\ 4 \\ -5 \end{pmatrix}$ $\begin{pmatrix} 5 \\ -3+s \\ p-3s \end{pmatrix} = \begin{pmatrix} 8+3t \\ 5+4t \\ -2-5t \end{pmatrix}$ $5 = 8 + 3t \Rightarrow t = -1$ $A = \begin{pmatrix} 8+3(-1) \\ 5+4(-1) \\ -2-5(-1) \end{pmatrix} = \begin{pmatrix} 5 \\ 1 \\ 3 \end{pmatrix} = (5,\ 1,\ 3)$ (b) $-3 + s = 5 + 4(-1) \Rightarrow s = 4$ $p - 3(4) = -2 - 5(-1) \Rightarrow p = 15$	M1 M1 A1 M1 A1	

Suka sampel ini?

Kertas Percubaan STPM

Sem 1 · Mathematics T

- Semester 1: kertas percubaan (trial) negeri, subjek yang dipilih + skema
- Blueprint 30 hari Semester 1 (setiap subjek)
- Semester 3: percubaan 2024, 10 subjek, banyak negeri, soalan + skema dalam satu fail

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