Section A [45 marks]

Answer **all** questions in this section.

1 The functions f and g are defined as follows:

$$f : x \mapsto x^2 - 6x,\ x \leq 3$$
$$g : x \mapsto 4 - e^{-x},\ x \in \mathbb{R}$$

(a) Show that f^{-1} exists. Find f^{-1} and state its domain and range. [4 marks]

(b) Show that composite function fg does not exist. Find the maximum domain of g such that fg exists. [3 marks]

2 The sum, S_n of the first n terms of a series is given by $S_n = kn + mn^2$.
Given that $S_2 = -3$ and $S_6 = 51$,

(a) find the values of k and m. [3 marks]

(b) find the nth term of the series. [2 marks]

(c) show that the series is an arithmetic progression and find its common difference. [2 marks]

3 Solve the following system of linear equations by using Gaussion elimination.

$$\begin{aligned} 2x + y - z &= 9 \\ x - 2y - 3z &= -3 \\ 3x + y + 4z &= 6 \end{aligned}$$

[6 marks]

4 The complex number z is given by $z = \sqrt{3} + i$.

(a) Find $|z|$ and $\arg z$. [3 marks]

(b) Using de Moivre's theorem, show that $z^4 = 8\left(-1 + \sqrt{3}i\right)$. [3 marks]

(c) Express $\frac{z^3}{z^*}$ in the form of $x + yi$, where z^* is the conjugate of z and $x, y \in \mathbb{R}$. [3 marks]

5 (a) Show that the equation $4x^2 - 9y^2 - 16x - 54y - 101 = 0$ represents a hyperbola. [2 marks]

(b) Find the coordinates of the centre, the vertices and the equations of the asymptotes. [4 marks]

(c) Sketch the hyperbola. [2 marks]

6 A straight line is defined by $\frac{x-1}{3} = \frac{y+2}{4} = 3 - z$ and a plane is defined by $2x - 3y + 4z = 40$.
The straight line intersects the plane at a point P.

(a) Find the coordinates of P. [4 marks]

(b) Determine the angle between the straight line and the plane. [4 marks]

Section B [15 marks]

Answer any **one** question in this section.

7 Express $12 \cos x + 9 \sin x$ in the form $r \cos (x - \alpha)$ where $r > 0$ and $0 < \alpha < \frac{\pi}{2}$. [4 marks]

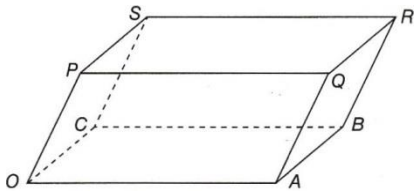
Hence,

(a) state the minimum and maximum values of $12 \cos x + 9 \sin x$ and find the corresponding values of x in the interval $0 \leq x \leq 2\pi$. [3 marks]

(b) solve the equation $12 \cos x + 9 \sin x = 0$ for $0 \leq x \leq 2\pi$. [3 marks]

(c) sketch the graph of $y = 12 \cos x + 9 \sin x$ for $0 \leq x \leq 2\pi$, and determine the range of values of x in this interval satisfying the inequality $0 \leq 12 \cos x + 9 \sin x \leq 10$. [5 marks]

8 The diagram below shows a parallelepiped. O is the origin. $OABC$, $PQRS$, $ABRQ$ and $OCSP$ are rectangles.



The unit vectors $\underset{\sim}{i}$ and $\underset{\sim}{j}$ are parallel to OA and OC respectively, and the unit vector $\underset{\sim}{k}$ is perpendicular to the plane $OABC$. The vectors $\overrightarrow{OA}$, $\overrightarrow{OB}$ and $\overrightarrow{OP}$ are $12\underset{\sim}{i}$, $12\underset{\sim}{i} + 5\underset{\sim}{j}$ and $2\underset{\sim}{i} + 4\underset{\sim}{k}$ respectively.

(a) Show that $\cos \angle BQS = \frac{49\sqrt{5}}{195}$. [6 marks]

(b) Calculate the area of the triangle AQS. [6 marks]

(c) Find the equation of the plane AQS. [3 marks]

MATHEMATICAL FORMULAE

Trigonometrical identities

- $\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$
- $\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$
- $\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$
- $\sin 2A = 2 \sin A \cos A$
- $\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$
- $\tan 2A = \dfrac{2 \tan A}{1 - \tan^2 A}$

Sums of series

- For an arithmetic series

$$S_n = \frac{1}{2}n(a + \ell) = \frac{1}{2}n[2a + (n-1)d]$$

- For a geometric series

$$S_n = \frac{a(1 - r^n)}{1 - r}, \quad r \neq 1$$

Binomial expansions

- $(a + b)^n = a^n + \binom{n}{1} a^{n-1}b + \binom{n}{2} a^{n-2}b^2 + \ldots + \binom{n}{r} a^{n-r}b^r + \ldots + b^n, \quad n \in \mathbb{Z}^+$
- $(1 + x)^n = 1 + nx + \dfrac{n(n-1)}{2!}x^2 + \ldots + \dfrac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots, \quad n \in \mathbb{Q}, \ |x| < 1$

Conics

- Parabola with vertex (h, k), focus $(a + h, k)$ and directrix $x = -a + h$

$$(y - k)^2 = 4a(x - h)$$

- Ellipse with centre (h, k) and foci $(-c + h, k)$, $(c + h, k)$, where $c^2 = a^2 - b^2$

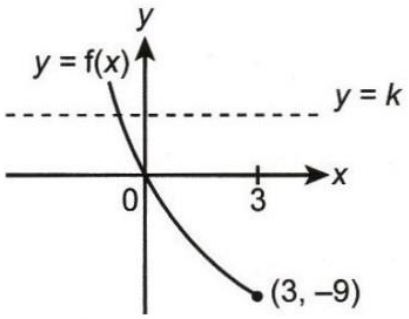
$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

- Hyperbola with centre (h, k) and foci $(-c + h, k)$, $(c + h, k)$, where $c^2 = a^2 + b^2$

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$

ANSWERS

1 (a) $f : x \mapsto x^2 - 6x = (x - 3)^2 - 9$



Any horizontal line $y = k$ is drawn horizontally with x-axis, will cut the graph at only one point.
Therefore f is a one-to-one function and f^{-1} exists. (shown)

Let $f^{-1}(x) = y$

$x = f(y)$

$x = (y - 3)^2 - 9$

$\pm\sqrt{x + 9} = y - 3$

$y = 3 - \sqrt{x + 9}$ as $y \leqslant 3$

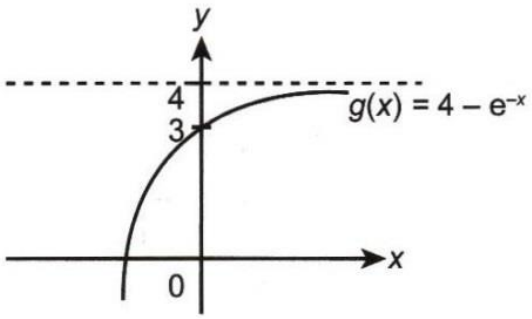
$f^{-1}(x) = 3 - \sqrt{x + 9}$

$f^{-1} : x \mapsto 3 - \sqrt{x + 9}$

$D_{f^{-1}} = [-9, \infty)$

$R_{f^{-1}} = (-\infty, 3]$ **4 marks**

(b)



$R_g = (-\infty, 4)$

$R_g = (-\infty, 4) \not\subset D_f = (-\infty, 3]$

Therefore fg does not exist. (shown)

We need $R_g = (-\infty, 3]$

$4 - e^{-x} \leqslant 3$

$x \leqslant 0$

Therefore the maximum domain of g for which fg exists $= (-\infty, 0]$ **3 marks**

[T : 7 marks]

2 (a) $S_2 = -3 \quad \Rightarrow \quad 2k + 4m = -3 \cdots ①$

$S_6 = 51 \quad \Rightarrow \quad 6k + 36m = 51 \cdots ②$

$① \times 3 : 6k + 12m = -9 \cdots ③$

$② - ③ : 24m = 60$

$$m = \frac{5}{2}$$

From ①: $2k + 4\left(\frac{5}{2}\right) = -3$

$$2k = -13$$

$$k = -\frac{13}{2}$$

3 marks

(b) $T_n = S_n - S_{n-1}$

$$= \left(-\frac{13}{2}n + \frac{5}{2}n^2\right) - \left[-\frac{13}{2}(n-1) + \frac{5}{2}(n-1)^2\right]$$

$$= -\frac{13}{2}n + \frac{5}{2}n^2 + \frac{13}{2}n - \frac{13}{2} - \frac{5}{2}n^2 + 5n - \frac{5}{2}$$

$$= 5n - 9$$

2 marks

(c) $T_n - T_{n-1} = (5n - 9) - [5(n-1) - 9]$

$$= 5n - 9 - 5n + 5 + 9$$

$$= 5$$

Hence, the series is an A.P where $d = 5$.

2 marks

[T : 7 marks]

3 $\left(\begin{array}{ccc|c} 2 & 1 & -1 & 9 \\ 1 & -2 & -3 & -3 \\ 3 & 1 & 4 & 6 \end{array}\right)$

$\xrightarrow{R1\leftrightarrow R2}\left(\begin{array}{ccc|c} 1 & -2 & -3 & -3 \\ 2 & 1 & -1 & 9 \\ 3 & 1 & 4 & 6 \end{array}\right)$

$\xrightarrow{R2-2R1\to R2}\left(\begin{array}{ccc|c} 1 & -2 & -3 & -3 \\ 0 & 5 & 5 & 15 \\ 3 & 1 & 4 & 6 \end{array}\right)$

$\xrightarrow{R3-3R1\to R3}\left(\begin{array}{ccc|c} 1 & -2 & -3 & -3 \\ 0 & 5 & 5 & 15 \\ 0 & 7 & 13 & 15 \end{array}\right)$

$\xrightarrow{\frac{1}{5}R2\to R2}\left(\begin{array}{ccc|c} 1 & -2 & -3 & -3 \\ 0 & 1 & 1 & 3 \\ 0 & 7 & 13 & 15 \end{array}\right)$

$\xrightarrow{R3-7R2\to R3}\left(\begin{array}{ccc|c} 1 & -2 & -3 & -3 \\ 0 & 1 & 1 & 3 \\ 0 & 0 & 6 & -6 \end{array}\right)$

$\xrightarrow{\frac{1}{6}R3\to R3}\left(\begin{array}{ccc|c} 1 & -2 & -3 & -3 \\ 0 & 1 & 1 & 3 \\ 0 & 0 & 1 & -1 \end{array}\right)$

$x-2y-3z=-3 \qquad .....(1)$

$y+z=3 \qquad .....(2)$

$z=-1 \qquad .....(3)$

Substitute $z=-1$ *into* (2), $\quad y=3-(-1)=4$

Substitute $y=4$ *&* $z=-1$ *into* (1), $\quad x=-3+2(4)+3(-1)=2$ **6 marks**

$\therefore \quad x=2,\ y=4,\ z=-1$ **[T : 6 marks]**

4 (a) $z = \sqrt{3} + i \qquad \Rightarrow |z| = \sqrt{3+1} = 2$

$$\Rightarrow \text{Arg } z = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$$

$$= \frac{\pi}{6}$$

3 marks

(b) $$z = 2\left[\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right]$$

$$z^4 = 2^4\left[\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right]^4$$

$$= 16\left[\cos\frac{4\pi}{6} + i\sin\frac{4\pi}{6}\right]$$

$$= 16\left[-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right]$$

$$= 8(-1 + \sqrt{3}i)$$

3 marks

(c) $$\frac{z^3}{z^*} = \frac{z^4}{zz^*}$$

$$= \frac{8(-1+\sqrt{3}i)}{(\sqrt{3}+i)(\sqrt{3}-i)}$$

$$= \frac{8(-1+\sqrt{3}i)}{3+1}$$

$$= 2(-1+\sqrt{3}i)$$

3 marks

[T : 9 marks]

Suka sampel ini?

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- Blueprint 30 hari Semester 1 (setiap subjek)
- Semester 3: percubaan 2024, 10 subjek, banyak negeri, soalan + skema dalam satu fail

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