2020-1-ME-MUNSYI ABDULLAH

Section A [45 *marks*]

Answer **all** *questions in this section*.

1 The polynomial p(x) = $6x^4 - ax^3 - bx^2 + 28x + 12$, where a and b are real constants, has factors ($x + 2$) and ($x - 2$).

(a) Find the values of a and b, and hence, factorise p(x) completely. [4m]

(b) Given that p(x) = $(2x - 3)[q(x) - 41 + 3x^3]$, find q($x$), and determine its range when $x \in [-2, 10]$. [2m]

2 Find the expansion of $\frac{1}{\sqrt{2-x}}$ in the ascending powers of x until the term in x^3. State the range of values of x for which this expansion is valid. [4m]

By taking $x = \frac{1}{5}$, find $\sqrt{10}$ correct to three decimal places. [3m]

3. A system of linear equations is given by

$x + 2y + z = q$, $2x + py + 4z = 3q$, $x + y + pz = q$.

Where p and q are constants.

(a) Write the augmented matrix for the system above and reduce it to row-echelon form. [4m)

(b) Determine the values of $p$and $q$such that the system has infinitely many solutions. Using the values $q$and the smaller value of p obtained, find the solutions of the system. [4m]

4 The complex number z is such that $z\bar{z} = 64$ and $\arg(z) = \frac{2\pi}{3}$.

(a) Find z in polar form and deduce z^5 in polar form. [3m]

(b) Find the complex numbers w such that

$w^3 = -4 + 4\sqrt{3}i$ /in the form of $a + b$i . [5m]

5. The equation of a hyperbola is $16x^2 - 9y^2 - 96x - 18y - 9 = 0.$

(a) Obtain the standard form for the equation of the hyperbola [2m]

(b) Find (i) the coordinates of the centre C,

(ii) the vertex V_1 and the vertex V_2,

(iii) the equations of the asymptote. [3m]

Sketch the hyperbola, and indicate the point C, V_1, V_2 and the asymptotes. [2m]

6. Relative to the origin $\boldsymbol{O}$, two points $\boldsymbol{A}$ and $\boldsymbol{B}$ have position vectors given by

$a = 14i + 14j + 14k$ *and* $b = 11i - 13j + 2k$ respectively.

(i) The point P divides the line AB in the ratio 2 :1. Find the coordinates of $\boldsymbol{P}$. [2m]

(ii) Show that $\boldsymbol{AB}$ and $\boldsymbol{OP}$ are perpendicular. [2m]

(iii) Find $a \times p$, where **p** is the vector $\vec{OP}$, and give the geometrical meaning of $|a \times p|$. Hence write down the area of triangle $\boldsymbol{OAP}$. [5m]

Section B [15 *marks*]

*Answer***one** *question only in this section.*

7. Given that $z^2 = \dfrac{2i}{(1+3i)^2}$.

(a) Find the real and imaginary parts of z^2. [4m]

Hence, obtain z_1 and z_2 which satisfy the above equation. [5m]

(b) Given that z_1 and z_3 are roots of $5x^2 + ax + b = 0$, where a and b are integers.

(i) Find the values of a and b. [3m]

(ii) Determine z_3, and deduce the relationship between z_1 and z_3. [3m]

8. (a) Show that $\dfrac{\sin\alpha + \sin\beta}{\cos\alpha - \cos\beta} = \cot\left(\dfrac{\beta - \alpha}{2}\right)$. [4m]

(b) Use trigonometric identities to verify that

(i) $\sin\theta = \dfrac{2\tan\frac{\theta}{2}}{1 + \tan^2\frac{\theta}{2}}$ [3m]

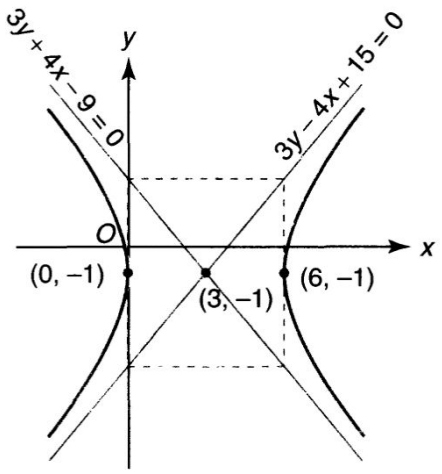
(ii) $\cos\theta = \dfrac{1 - \tan^2\frac{\theta}{2}}{1 + \tan^2\frac{\theta}{2}}$. [3m]

1(a)	$p(x) = 6x^4 - ax^3 - bx^2 + 28x + 12$ $= (x+2)(x-2)(6x^2 + kx - 3)$ $= (x^2 - 4)(6x^2 + kx - 3)$ Coefficient x : $28 = -4k \Rightarrow k = -7$ Coefficient x^3 : $-a = k \Rightarrow a = 7$ Coefficient x^2 : $-bx = -3 - 24 \Rightarrow b = 27$ $\therefore p(x) = (x+2)(x-2)(6x^2 + kx - 3)$ $= (x+2)(x-2)(3x+1)(2x-3)$	M1 M1 A1 A1	Quadratic factor seen Comparing coeffiecient COA of a & b Factorize completely
(b)	$p(x) = (2x-3)[q(x) - 41 + 3x^3]$ $q(x) - 41 + 3x^3 = (x^2 - 4)(3x+1)$ $= 3x^3 + x^2 - 12x - 4$ $q(x) = x^2 - 12x + 37$ $q(x) = (x-6)^2 + 1$ $\therefore 1 \le q(x) \le 65$	M1 A1	Sub x=-2 and 10
2	$\frac{1}{\sqrt{2-x}} = (2-x)^{-\frac{1}{2}}$ $\frac{1}{\sqrt{2-x}} = 2^{-\frac{1}{2}}(1-\frac{x}{2})^{-\frac{1}{2}}$ $\frac{1}{\sqrt{2-x}} = 2^{-\frac{1}{2}}[1 - \frac{1}{2}(-\frac{x}{2}) + \frac{(-\frac{1}{2})(-\frac{3}{2})}{(1)(2)}(-\frac{x}{2})^2 + \frac{(-\frac{1}{2})(-\frac{3}{2})(-\frac{5}{2})}{(1)(2)(3)}(-\frac{x}{2})^3 + ...]$ $\frac{1}{\sqrt{2-x}} = 2^{-\frac{1}{2}}[1 + \frac{1}{4}x + \frac{3}{32}x^2 + \frac{5}{128}x^3 + ...]$ the range of values of x for this expansion is valid is $\{x \mid x \in R, -2 < x < 2\}$ Subst. $x = \frac{1}{5}$, $\frac{1}{\sqrt{2-\frac{1}{5}}} = 2^{-\frac{1}{2}}[1 + \frac{1}{4}(\frac{1}{5}) + \frac{3}{32}(\frac{1}{5})^2 + \frac{5}{128}(\frac{1}{5})^3 + ...]$ $\frac{1}{\sqrt{\frac{9}{5}}} = \frac{1}{\sqrt{2}}(1.05406)$ $\frac{\sqrt{5}}{3} = \frac{1}{\sqrt{2}}(1.05406)$ $\sqrt{10} = 3(1.05406)$ $\sqrt{10} \approx 3.162$ (correct to 3 decimal places)	B1 M1 A1 B1 M1 B1 A1	CAO Sub $x = \frac{1}{5}$ into his expension ,provided earlier M1 obtained

3 (a)	$(q\ 3q\ q\ )$ $R_3 - R_1 \to R_3$ $R_2 - 2R_1 \to R_2$ $(q\ q\ 0\ )$ $(p-4)R_3 + R_2 \to R_3$ $(q\ q\ q\ )$ $p = 2, 3$ *and* $q = 0$ $z = t, \quad (p-4)y + 2t = 0$ *smaller value of* $p = 2$	B1 M1 M1 A1	
(b)	$(2-4)y + 2t = 0$ $-2y + 2t = 0$ $y = t$ $z = t, \quad y = t$ *into* $x + 2y + z = 0$ $x = 3t$	B1 M1 M1 A1	Substitute $z=t$ Choosing smaller value of p

4(a)	$z\bar{z} = \lvert z\rvert^2 = 64$ $\lvert z\rvert = 8$ $z = 8\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$ $z^5 = 8^5 \operatorname{cis}\left(\frac{10\pi}{3}\right)$ $= 32768\operatorname{cis}\left(-\frac{2\pi}{3}\right)$ $= 32768(\cos cos\ (-\frac{2\pi}{3})\ + sin\ (-\frac{2\pi}{3}))$	B1 M1 A1	Express in polar form Applied de Moivre's theorem
(b)	$\lvert -4 + 4\sqrt{3}i\rvert = \sqrt{4^2 + (4\sqrt{3})^2} = 8$ $\arg(-4 + 4\sqrt{3}i) = \pi - \tan^{-1}\left(\frac{4\sqrt{3}}{4}\right) = \frac{2\pi}{3}$ $w^3 = 8\operatorname{cis}\left(\frac{2\pi}{3}\right)$	M1 A1 M1 M1	

	$w = 2\text{cis}\left(\frac{2\pi/3 + 2k\pi}{3}\right), k = 0,1,2$ $w_1 = 2\text{cis}\left(\frac{2\pi}{9}\right), w_2 = 2\text{cis}\left(\frac{8\pi}{9}\right), w_3 = 2\text{cis}\left(\frac{14\pi}{9}\right)$ $w_1 = 2\text{cis}\left(\frac{2\pi}{9}\right), w_2 = 2\text{cis}\left(\frac{8\pi}{9}\right), w_3 = 2\text{cis}\left(-\frac{4\pi}{9}\right)$ $w = 1.5321 + 1.2856i, -1.8794 + 0.6840i, 0.3473 - 1.9696i$	A1	

5.(a)	$16(x^2 - 6x) - 9(y^2 + 2y) = 9$ $16(x-3)^2 - 9(y+1)^2 = 144$ $\frac{(x-3)^2}{3^2} - \frac{(y+1)^2}{4^2} = 1$ or $\frac{(x-3)^2}{9} - \frac{(y+1)^2}{16} = 1$	M1 A1	Seen $16(x-3)^2$ or $9(y+1)^2$ CAO
5.(b)	(i) $C: (3,-1)$ (ii) $V_1: (0,-1)$ and $V_2: (6,-1)$ (iii) Asymptotes : $3y - 4x + 15 = 0$, $3y + 4x - 9 = 0$	A1 A1 A1	
5.(c)		D1 D1	Hyperbola open right-left centred at fourth quardrant Centre, vertices and asymptotes correctly shown and labelled

6(i)

(b)	$Z^2 = \frac{2i}{(1+3i)^2} = \frac{2i}{(1+3i)^2} \bullet \frac{(1-3i)^2}{(1-3i)^2}$ $= \frac{2i(1-9-6i)}{(1+9)^2} = \frac{12-16i}{100}$ $\therefore Re\ (Z^2) = \frac{3}{25} \qquad Im\ (Z^2) = -\frac{4}{25}$ *Let* $Z = p + qi \ \ (p, q \in R)$: $\therefore (p^2 + q^2) + 2pqi = \frac{3}{25} - \frac{4}{25}i$ $p^2 - q^2 = \frac{3}{25} \ldots\ldots (1)$ *and* $2pqi = -\frac{4}{25}i \ldots\ldots (2)$ $p^2 - \frac{4}{625p^2} = \frac{3}{25}$ $p^4 - \frac{3}{25}p^2 - \frac{4}{625} = 0$ $(p^2 + \frac{1}{25})(p^2 - \frac{4}{25}) = 0$ Since $p^2 > 0 \qquad \therefore\ p^2 = \frac{4}{25}$ $p = \pm\frac{2}{5}$ $\therefore Z_1 = \frac{2}{5} - \frac{1}{5}i \qquad Z_2 = -\frac{2}{5} + \frac{1}{5}i$ Z_1 *is a root* : $5Z_1{}^2 + a\,Z_1 + b = 0$ (i) $\therefore 5\ (\frac{3}{25} - \frac{4}{25}i)\ + a\ (\frac{2}{5} - \frac{1}{5}i)\ + b = 0$ $\frac{3}{5} + \frac{2}{5}a + b = 0$ *and* $-\frac{4}{5} - \frac{1}{5}a = 0$ $\therefore a = -4, \quad b = 1$ (ii) SOR : $Z_1 + Z_3 = -\frac{a}{5}$ $\therefore Z_3 = \frac{4}{5} - (\frac{2}{5} - \frac{1}{5}i) = \frac{2}{5} + \frac{1}{5}i$ $\therefore Z_1$ *and* Z_3 *are a pair of conjugates .* $(Z_1 = Z_3{}^*)$	M1 M1 A1A1 M1 M1 A1A1 M1 M1 A1 M1 M1 A1	Multiply with conjugate Simplified Comparing his Re and Im part Eliminating p or q Factorize and choose $p^2 > 0$ Substitute Z_1 into given equation Equating Re and Im part

8 (a)	$\frac{\sin\alpha + \sin\beta}{\cos\alpha - \cos\beta} = \frac{2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)}{-2\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)}$ $= \frac{\cos\left(\frac{\alpha-\beta}{2}\right)}{-\sin\left(\frac{\alpha-\beta}{2}\right)}$ $= -\cot\left(\frac{\alpha-\beta}{2}\right)$ $= \cot\left(\frac{\beta-\alpha}{2}\right)$	M1 M1 M1 A1	

Alternative

(b)(i)

$$\sin\theta = 2\sin\frac{\theta}{2}\cos\frac{\theta}{2}$$

$$= 2\sin\frac{\theta}{2}\cos\frac{\theta}{2}\left(\frac{\cos^2\frac{\theta}{2}}{\cos^2\frac{\theta}{2}}\right)$$ M1

$$= \frac{2\tan\frac{\theta}{2}}{\sec^2\frac{\theta}{2}}$$

$$= \frac{2\tan\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}}$$ M1

(ii)

$$\frac{2\tan\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}} = \frac{2\dfrac{\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}}}{1+\dfrac{\sin^2\frac{\theta}{2}}{\cos^2\frac{\theta}{2}}}$$ A1

$$= \frac{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{\cos^2\frac{\theta}{2}+\sin^2\frac{\theta}{2}}$$ M1

$$= \sin\theta$$ M1

$$\cos\theta = 1-2\sin^2\frac{\theta}{2}$$

$$= \left(1-2\sin^2\frac{\theta}{2}\right)\left(\frac{\cos^2\frac{\theta}{2}}{\cos^2\frac{\theta}{2}}\right)$$ A1

$$= \frac{\sec^2\frac{\theta}{2}-2\tan^2\frac{\theta}{2}}{\sec^2\frac{\theta}{2}}$$

$$= \frac{\left(\tan^2\frac{\theta}{2}+1\right)-2\tan^2\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}}$$

$$= \frac{1-\tan^2\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}}$$

Altenative

$$\frac{1-\tan^2\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}} = \frac{1-\frac{\sin^2\frac{\theta}{2}}{\cos^2\frac{\theta}{2}}}{1+\frac{\sin^2\frac{\theta}{2}}{\cos^2\frac{\theta}{2}}}$$

M1

$$= \cos^2\frac{\theta}{2} - \sin^2\frac{\theta}{2}$$

M1

$$= \cos 2\left(\frac{\theta}{2}\right)$$

$$= \cos\theta$$

$3\sin\theta + \cos\theta = 2$

A1

$$3\left(\frac{2\tan\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}}\right) + \left(\frac{1-\tan^2\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}}\right) = 2$$

M1

sub $\tan\frac{\theta}{2} = t$

A1

$$3\left(\frac{2t}{1+t^2}\right) + \left(\frac{1-t^2}{1+t^2}\right) = 2$$

$3t^2 - 6t + 1 = 0$

$t = \frac{1}{3}(3 \pm \sqrt{6})$

$= 0.1835 @ 1.8165$

$\tan\frac{\theta}{2} = t$

$\frac{\theta}{2} = \tan^{-1}(0.1835)$ $\quad$ $\frac{\theta}{2} = \tan^{-1}(1.8165)$

$\theta = 20.796°$ $\quad$ $\theta = 122.334°$

Suka sampel ini?

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Sem 1 · Mathematics T

- Semester 1: kertas percubaan (trial) negeri, subjek yang dipilih + skema
- Blueprint 30 hari Semester 1 (setiap subjek)
- Semester 3: percubaan 2024, 10 subjek, banyak negeri, soalan + skema dalam satu fail

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