2020-1-NS-ST PAUL

Section A [45 *marks*]

Answer **all** *questions*

1. Express $8 \sin \theta - 15 \cos \theta$ in the form $R \sin (\theta - \alpha)$, where $R > 0$ and $0° < \alpha < 90°$, giving exact value of R and the value of α correct to two decimal places. Hence, solve the equation

$$8 \sin \theta - 15 \cos \theta = 14$$

giving all solutions in the interval $0° \leq \theta \leq 360°$. [6]

2. (a) Use binomial expansions to show $\sqrt{\frac{1+4x}{1-x}} \approx 1 + \frac{5}{2}x - \frac{5}{8}x^2$. [4]

(b) A student substitutes $x = \frac{1}{2}$ into both sides of the approximations as shown in part (a) in an attempt to find an approximation to $\sqrt{6}$. Give a reason why the student should not use $x = \frac{1}{2}$. [1]

(c) Substitute $x = \frac{1}{11}$ into $\sqrt{\frac{1+4x}{1-x}} \approx 1 + \frac{5}{2}x - \frac{5}{8}x^2$ to obtain an approximation to $\sqrt{6}$. Give your answer as a fraction in its simplest form. [3]

3. A system of linear equations is given by

$$2x - 3y + 4z = 1$$
$$3x - y = 2$$
$$x + 2y + kz = 1$$

where k is a real constant.

(a) Reduce the augmented matrix of the system of linear equations to row-echelon form. [4]

(b) State the value of k such that the system has no unique solution. For this value of k, solve the system of equations. [4]

4. Express the complex number $-\frac{1}{2} + \frac{\sqrt{3}}{2}i$ in polar form. [2]

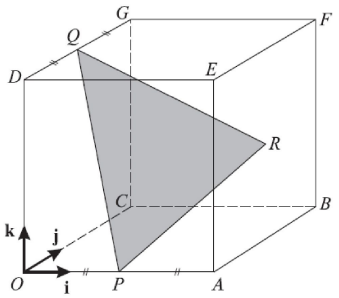
Hence, using de Moivre's theorem, solve the equation $(z + \frac{1}{2}i)^4 = -\frac{1}{2} + \frac{\sqrt{3}}{2}i$. Give your answers in cartesian form. [4]

5. The equation of a conic is $5x^2 + 4y^2 - 40x + 8y + 64 = 0$.

(a) Obtain the standard form of its equation and identify the conic. [3]

(b) Find the coordinates of the centre, vertices and foci. Hence, sketch the conic. [5]

6.



The diagram shows a cube OABCDEFG in which the length of each side is 4 units. The unit vectors **i**, **j**, **k** are parallel to $\overrightarrow{OA}\ \overrightarrow{OA}$, $\overrightarrow{OC}\ \overrightarrow{OC}$ and $\overrightarrow{OD}\ \overrightarrow{OD}$ respectively. The mid-points of OA and DG are P and Q respectively and R is the centre of the square face ABFE.

(a) Express each of the vectors $\overrightarrow{PR}\ \overrightarrow{PR}$ and $\overrightarrow{PQ}\ \overrightarrow{PQ}$ in terms of **i**, **j** and **k**. [2]
(b) Find the angle QPR. [3]
(c) Find the area of the triangle PQR. [4]

Section B [15 marks]

*Answer **one** question only*

7. Matrix A is given by A = $\begin{pmatrix}1 & 0 & 2\\3 & 1 & 6\\8 & 9 & 2\end{pmatrix}\begin{pmatrix}1 & 0 & 2\\3 & 1 & 6\\8 & 9 & 2\end{pmatrix}$.

(a) Find the determinant of matrix A. Deduce $\begin{vmatrix}0 & 1 & 2\\1 & 3 & 6\\9 & 8 & 2\end{vmatrix}\begin{vmatrix}0 & 1 & 2\\1 & 3 & 6\\9 & 8 & 2\end{vmatrix}$. [5]
(b) By performing elementary row operations on the augmented matrix (A|I), where I is the 3×3 identity matrix, find A^{-1}. [6]
(c) Hence, solve the simultaneous equation

$3x + y + 6z = 2$
$8x + 9y + 2z = -1$
$x + 2z = 1$ [4]

8. The plane π passes through the points with coordinates A (4, -1, -3) and B (-2, -5, 2) and is parallel to the vector $3\mathbf{i} + \mathbf{j} - 2\mathbf{k}$. The line l_1 has equation $\frac{x-1}{2} = \frac{y-2}{-4} = \frac{z+3}{1}$ and l_2 has equation $\frac{x+2}{1} = \frac{y-1}{5} = \frac{z-3}{p}$ where p is a constant.

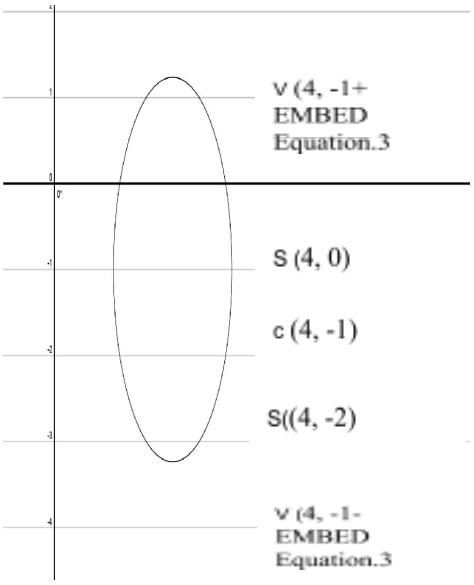
(a) Find the cartesian equation of the plane π. [4]
(b) It is given that l_1 and l_2 intersect. Show that p = -7 [4]
(c) Find the coordinates of the point at which the line l_2 intersects the plane π. [4]
(d) Find the acute angle between the line l_2 and the plane π. [3]

2020-1-NS-ST PAUL-MARKIING SCHEME

No.	Scheme	M	Remarks
1	$8\sin\theta - 15\cos\theta \equiv R\sin(\theta-\alpha)$ $\equiv R\sin\theta\cos\alpha - R\cos\theta\sin\alpha$ $R\cos\alpha = 8$ ---(1) $\quad R\sin\alpha = 15$ ---(2)	1	Either one equa. correct
	$(1)^2 + (2)^2$, $R^2 = 8^2 + 15^2$ $\quad$ (2) ÷ (1), $\dfrac{R\sin\alpha}{R\cos\alpha} = \dfrac{15}{8}$ $R = 17$ $\quad\quad$ $\alpha = 61.93°$	1 1	Both R and α correct CAO
	$\therefore 8\sin\theta - 15\cos\theta \equiv 17\sin(\theta - 61.93°)$		
	$8\sin\theta - 15\cos\theta = 14$, $0° \le \theta \le 360°$ $17\sin(\theta - 61.93°) = 14$	1	
	$\sin(\theta - 61.93°) = \dfrac{14}{17}$ $\theta - 61.93° = 55.44°$, $124.56°$ $\theta = 117.37°$, $186.49°$	1 1	Either one correct Both correct
2	(a) $\sqrt{\dfrac{1+4x}{1-x}} = (1+4x)^{\frac{1}{2}}(1-x)^{-\frac{1}{2}}$		
	$(1+4x)^{\frac{1}{2}} = 1 + \dfrac{1}{2}(4x) + \dfrac{\frac{1}{2}(-\frac{1}{2})}{2!}(4x)^2 + \ldots$ $= 1 + 2x - 2x^2 + \ldots$	1	
	$(1-x)^{-\frac{1}{2}} = 1 + (-\dfrac{1}{2})(-x) + \dfrac{(-\frac{1}{2})(-\frac{3}{2})}{2!}(-x)^2 + \ldots$ $= 1 + \dfrac{x}{2} + \dfrac{3}{8}x^2 + \ldots$	1 1	
	$(1+4x)^{\frac{1}{2}}(1-x)^{-\frac{1}{2}} = (1 + 2x - 2x^2 + \ldots)\left(1 + \dfrac{x}{2} + \dfrac{3}{8}x^2 + \ldots\right)$ $= 1 + \dfrac{x}{2} + \dfrac{3}{8}x^2 + 2x + x^2 - 2x^2 + \ldots$ $\approx 1 + \dfrac{5}{2}x - \dfrac{5}{8}x^2$	1	His exp. of $(1+4x)^{\frac{1}{2}}$ × his exp of $(1-x)^{-\frac{1}{2}}$
	(b) $(1+4x)^{\frac{1}{2}}(1-x)^{-\frac{1}{2}}$ is valid for $-\dfrac{1}{4} < x < \dfrac{1}{4}$. Students should not use $x = \dfrac{1}{2}$ as $\dfrac{1}{2} > \dfrac{1}{4}$	1	
	(c) Let $x = \dfrac{1}{11}$,	1	Subs. $\dfrac{1}{11}$ to both sides

	$\sqrt{\dfrac{1+4\ (\frac{1}{11})}{1-(\frac{1}{11})}} \approx 1+\dfrac{5}{2}(\dfrac{1}{11})-\dfrac{5}{8}(\dfrac{1}{11})^2$ $\sqrt{\dfrac{\frac{15}{11}}{\frac{10}{11}}} \approx \dfrac{1183}{968}$ $\sqrt{\dfrac{3}{2}} \approx \dfrac{1183}{968}$ $\sqrt{\dfrac{3}{2}} \times 2 \approx \dfrac{1183}{968} \times 2$ $\sqrt{6} \approx \dfrac{1183}{484}$	1 1	CAO
3	(a) $\left(\begin{array}{ccc\|c} 2 & -3 & 4 & 1 \\ 3 & -1 & 0 & 2 \\ 1 & 2 & k & 1 \end{array}\right)$ $\xrightarrow{3R_1-2R_2\to R_2} \left(\begin{array}{ccc\|c} 2 & -3 & 4 & 1 \\ 0 & -7 & 12 & -1 \\ 1 & 2 & k & 1 \end{array}\right)$ $\xrightarrow{R_1-2R_3\to R_3} \left(\begin{array}{ccc\|c} 2 & -3 & 4 & 1 \\ 0 & -7 & 12 & -1 \\ 0 & -7 & 4-2k & -1 \end{array}\right)$ $\xrightarrow{R_2-R_3\to R_3} \left(\begin{array}{ccc\|c} 2 & -3 & 4 & 1 \\ 0 & -7 & 12 & -1 \\ 0 & 0 & 8+2k & 0 \end{array}\right)$ (b) $8 + 2k = 0$ $k = -4$ $2x - 3y + 4z = 1 \Rightarrow 6x - 9y + 12z = 3$ ---(1) $-7y + 12z = -1$ ---(2) (1) – (2), $6x - 2y = 4$ $y = 3x - 2$ let $x = t$, $y = 3t - 2$ $-7(3t - 2) + 12z = -1$ $z = \dfrac{7t - 5}{4}$ $\therefore x = t,\ y = 3t - 2,\ z = \dfrac{7t - 5}{4}$	1 1 1 1 1 1 1 1	Obtain 0 at (2,1) and (3,1) Obtain 0 at (3,2) REF Attempt to find y with his "let" Attempt to find z with his "let"

4	Let w = $-\frac{1}{2}+\frac{\sqrt{3}}{2}i$		
	$\lvert w\rvert = \sqrt{(-\frac{1}{2})^2+(\frac{\sqrt{3}}{2})^2} = 1 \quad \theta = \tan^{-1}\frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \frac{\pi}{3}$	1	Either |w| or arg (W) correct
	$\arg w = \frac{2\pi}{3}$		
	$w = 1(\cos\frac{2\pi}{3}+i\sin\frac{2\pi}{3})$	1	
	$(z+\frac{1}{2}i)^4 = -\frac{1}{2}+\frac{\sqrt{3}}{2}i$		
	$(z+\frac{1}{2}i) = (-\frac{1}{2}+\frac{\sqrt{3}}{2}i)^{\frac{1}{4}}$		
	$= [1(\cos\frac{2\pi}{3}+i\sin\frac{2\pi}{3})]^{\frac{1}{4}}$		Apply DM theorem on his polar form
	$= 1(\cos\frac{\frac{2\pi}{3}+2k\pi}{4}+i\sin\frac{\frac{2\pi}{3}+2k\pi}{4})$, k = 0, 1, 2, 3	1	Sub. with n = 4 and k = 0, 1, 2, 3
	k = 0, $z+\frac{1}{2}i = 1(\cos\frac{2\pi}{12}+i\sin\frac{2\pi}{12}) = \frac{\sqrt{3}}{2}+\frac{1}{2}i$	1	
	$\therefore$ z = $\frac{\sqrt{3}}{2}$		At least 2 terms correct (in polar form)
	k = 1, $z+\frac{1}{2}i = 1(\cos\frac{2\pi}{3}+i\sin\frac{2\pi}{3}) = -\frac{1}{2}+\frac{\sqrt{3}}{2}i$	1	
	$\therefore$ z = $-\frac{1}{2}+(\frac{\sqrt{3}}{2}-\frac{1}{2})i$		
	k = 2, $z+\frac{1}{2}i = 1(\cos\frac{7\pi}{6}+i\sin\frac{7\pi}{6}) = -\frac{\sqrt{3}}{2}-\frac{1}{2}i$		
	$\therefore$ z = $-\frac{\sqrt{3}}{2}-i$		
	k = 3, $z+\frac{1}{2}i = 1(\cos\frac{5\pi}{3}+i\sin\frac{5\pi}{3}) = \frac{1}{2}-\frac{\sqrt{3}}{2}i$		All answers in cartesian form correct
	z = $\frac{1}{2}-(\frac{\sqrt{3}}{2}+\frac{1}{2})i$		
		1	
5	(a) $5x^2 + 4y^2 - 40x + 8y + 64 = 0$ $5(x^2 - 8x) + 4(y^2 + 2y) = -64$ $5[(x-4)^2 - (-4)^2] + 4[(y+1)^2 - (1)^2] = -64$ $5(x-4)^2 + 4(y+1)^2 = 20$	1	

	$\frac{(x-4)^2}{4}+\frac{(y+1)^2}{5}=1$ ∴ It is an ellipse (b) Centre: (4, -1) c = 1 Vertices : (4, -1+ $\sqrt{5}$) , (4, -1 - $\sqrt{5}$) Foci: (4, -2), (4, 0)  	1 1 1 1 1 1 1	 Both Both Vertical ellipse passing 1st and 4th quad. with centre in 4th quad. All correct (centre, vertices and foci indicated)
6	(a) $\overrightarrow{PR}=\overrightarrow{PR}=$ $2\mathbf{i}+2\mathbf{j}+2\mathbf{k}$ $\overrightarrow{PQ}$ $\overrightarrow{PQ}$ $=-2\mathbf{i}+2\mathbf{j}+4\mathbf{k}$ (b) $\cos\angle QPR=\frac{\begin{pmatrix}-2\\2\\4\end{pmatrix}\cdot\begin{pmatrix}2\\2\\2\end{pmatrix}}{\sqrt{(-2)^2+2^2+4^2}\sqrt{2^2+2^2+2^2}}$ $\frac{\begin{pmatrix}-2\\2\\4\end{pmatrix}\cdot\begin{pmatrix}2\\2\\2\end{pmatrix}}{\sqrt{(-2)^2+2^2+4^2}\sqrt{2^2+2^2+2^2}}$ $\angle QPR=\cos^{-1}\frac{8}{\sqrt{24}\sqrt{12}}$ $=61.87°$ (c) $\overrightarrow{PQ}\times\overrightarrow{PR}\ \overrightarrow{PQ}\times\overrightarrow{PR}=\begin{vmatrix}\boldsymbol{i}&\boldsymbol{j}&\boldsymbol{k}\\-2&2&4\\2&2&2\end{vmatrix}\begin{vmatrix}\boldsymbol{i}&\boldsymbol{j}&\boldsymbol{k}\\-2&2&4\\2&2&2\end{vmatrix}$ $=-4\mathbf{i}+12\mathbf{j}-8\mathbf{k}$ Area of $\Delta PQR\ \Delta PQR = \frac{1}{2}\lvert\overrightarrow{PQ}\times\overrightarrow{PR}\rvert=\frac{1}{2}\sqrt{(-4)^2+(12)^2+(-8)^2}$ $=2\sqrt{14}$ units^2	1 1 1 1 1 1 1 1 1	

7	(a) $\lvert A\rvert = 1\begin{vmatrix}1 & 6\\ 9 & 2\end{vmatrix} - 0\begin{vmatrix}3 & 6\\ 8 & 2\end{vmatrix} + 2\begin{vmatrix}3 & 1\\ 8 & 9\end{vmatrix}$	1 1 1	
	$\begin{vmatrix}1 & 6\\ 9 & 2\end{vmatrix} - 0\begin{vmatrix}3 & 6\\ 8 & 2\end{vmatrix} + 2\begin{vmatrix}3 & 1\\ 8 & 9\end{vmatrix}$		
	$= 1(2 - 54) - 0(6 - 48) + 2(27 - 8)$ $= -14$	1	Opposite sign with his det.
		1	CAO
	$\begin{vmatrix}0 & 1 & 2\\ 1 & 3 & 6\\ 9 & 8 & 2\end{vmatrix} = -(-14)\begin{vmatrix}0 & 1 & 2\\ 1 & 3 & 6\\ 9 & 8 & 2\end{vmatrix} = -(-14)$	1	
	$= 14$		
	(b) $(A\lvert I) = \left(\begin{array}{ccc\lvert ccc}1 & 0 & 2 & 1 & 0 & 0\\ 3 & 1 & 6 & 0 & 1 & 0\\ 8 & 9 & 2 & 0 & 0 & 1\end{array}\right)$ $\left(\begin{array}{ccc\lvert ccc}1 & 0 & 2 & 1 & 0 & 0\\ 3 & 1 & 6 & 0 & 1 & 0\\ 8 & 9 & 2 & 0 & 0 & 1\end{array}\right)$	1	Obtain 0 at (2,1) and (3,1) Obtain 0 at (1,2) and (3,2)
	$\xrightarrow{\substack{R_2-3R_1\to R_2\\ R_3-8R_1\to R_3}}$ $\xrightarrow{\substack{R_2-3R_1\to R_2\\ R_3-8R_1\to R_3}}$ $\left(\begin{array}{ccc\lvert ccc}1 & 0 & 2 & 1 & 0 & 0\\ 0 & 1 & 0 & -3 & 1 & 0\\ 0 & 9 & -14 & -8 & 0 & 1\end{array}\right)$		
	$\left(\begin{array}{ccc\lvert ccc}1 & 0 & 2 & 1 & 0 & 0\\ 0 & 1 & 0 & -3 & 1 & 0\\ 0 & 9 & -14 & -8 & 0 & 1\end{array}\right)$		Obtain identity matrix at left side All correct
	$\xrightarrow{R_3-9R_2\to R_3}$ $\xrightarrow{R_3-9R_2\to R_3}$ $\left(\begin{array}{ccc\lvert ccc}1 & 0 & 2 & 1 & 0 & 0\\ 0 & 1 & 0 & -3 & 1 & 0\\ 0 & 0 & -14 & 19 & -9 & 1\end{array}\right)$	1	CAO
	$\left(\begin{array}{ccc\lvert ccc}1 & 0 & 2 & 1 & 0 & 0\\ 0 & 1 & 0 & -3 & 1 & 0\\ 0 & 0 & -14 & 19 & -9 & 1\end{array}\right)$	1	
	$\xrightarrow{-\frac{1}{14}R_3\to R_3}$ $\xrightarrow{-\frac{1}{14}R_3\to R_3}$ $\left(\begin{array}{ccc\lvert ccc}1 & 0 & 2 & 1 & 0 & 0\\ 0 & 1 & 0 & -3 & 1 & 0\\ 0 & 0 & 1 & -\frac{19}{14} & \frac{9}{14} & -\frac{1}{14}\end{array}\right)$	1	Multiply with his inverse of A
	$\left(\begin{array}{ccc\lvert ccc}1 & 0 & 2 & 1 & 0 & 0\\ 0 & 1 & 0 & -3 & 1 & 0\\ 0 & 0 & 1 & -\frac{19}{14} & \frac{9}{14} & -\frac{1}{14}\end{array}\right)$	1	
	$\xrightarrow{R_1-2R_3\to R_1}$ $\left(\begin{array}{ccc\lvert ccc}1 & 0 & 0 & \frac{26}{7} & -\frac{9}{7} & \frac{1}{7}\\ 0 & 1 & 0 & -3 & 1 & 0\\ 0 & 0 & 1 & -\frac{19}{14} & \frac{9}{14} & -\frac{1}{14}\end{array}\right)$	1 1	
	$\xrightarrow{R_1-2R_3\to R_1}$ $\left(\begin{array}{ccc\lvert ccc}1 & 0 & 0 & \frac{26}{7} & -\frac{9}{7} & \frac{1}{7}\\ 0 & 1 & 0 & -3 & 1 & 0\\ 0 & 0 & 1 & -\frac{19}{14} & \frac{9}{14} & -\frac{1}{14}\end{array}\right)$		

Suka sampel ini?

Kertas Percubaan STPM

Sem 1 · Mathematics T

- Semester 1: kertas percubaan (trial) negeri, subjek yang dipilih + skema
- Blueprint 30 hari Semester 1 (setiap subjek)
- Semester 3: percubaan 2024, 10 subjek, banyak negeri, soalan + skema dalam satu fail

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