SAMPEL PERCUMA

Trial STPM Sem 1 Mathematics T Perak

Set penuh: Kertas Percubaan STPM

Sem 1 · Mathematics T

Sampel ini: 7 muka surat. Set penuh ada:

- 3 fail PDF
- Semester 1: kertas percubaan (trial) negeri, subjek yang dipilih + skema
- Blueprint 30 hari Semester 1 (setiap subjek)
- Semester 3: percubaan 2024, 10 subjek, banyak negeri, soalan + skema dalam satu fail

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Sampel bermula di muka surat sebelah

Section A [45 *marks*]

Answer **all** *questions.*

1. Express $\cos x + \sin x$ in the form $r\cos(x - \alpha)$, where $r > 0$ and $0 < \alpha < \frac{1}{2}\pi$.
   State the maximum and minimum of $\cos x + \sin x$. [4 *marks*]

   (a) Sketch the graph of $y = |\cos x + \sin x|$ for $0 \le x \le 2\pi$. [2 *marks*]

   (b) Solve the equation $|\cos x + \sin x| = 1$ for $0 \le x \le 2\pi$.
   Hence, use your graph in (a) to find the set of values of x in this interval satisfying the inequality $|\cos x + \sin x| > 1$. [4 *marks*]

2. A sequence $u_1, u_2, u_3, \ldots$ is defined by the recursive formula $u_{n+1} = 1 + \frac{1}{2}u_n$ with $u_1 = 1$

   (a) Write down each of the terms u_2, u_3 and u_4 in the form of $\sum_{r=0}^{n-1} \left(\frac{1}{2}\right)^r$. [3 *marks*]

   (b) Show that the explicit formula for u_n is given by $u_n = 2\left[1 - \left(\frac{1}{2}\right)^n\right]$.
   Deduce $\lim_{n\to\infty} u_n$ and state, with a reason, whether the sequence $u_1, u_2, u_3, \ldots$ is a convergent. [5 *marks*]

3. A system of linear equations is given by

$$x + 3y + 2z = k$$
$$x - y + z = 2$$
$$2x + 6y + hz = -4$$

where h and k are constants.

(a) Use elementary row operation, show that the augmented matrix of the system of linear equation may be reduced to row echelon form

$$\left(\begin{array}{ccc|c} 1 & 3 & 2 & k \\ 0 & 4 & 1 & k-2 \\ 0 & 0 & 4-h & 2k+4 \end{array}\right)$$

[4 *marks*]

(b) Find the values of h and k such that the system has
   (i) no solution, [1 *mark*]
   (ii) infinitely many solutions, [1 *mark*]
   (iii) a unique solution. [1 *mark*]

4. Find the polar form of the complex number $2 - i\sqrt{12}$. [3 *marks*]
Using De Moivre's theorem, solve the equation $z^5 = 2 - i\sqrt{12}$ giving your answer in the Cartesian form $x + iy$. [5 *marks*]

5. The parametric equations of a conic are $x = 3 - \cos 2\theta$ and $y = 4 \sin \theta$ where $0 \leq \theta \leq 2\pi$.

(a) Show that the standard form for the equation of the conic is $y^2 = 8(x - 2)$ and state the type of conic. [3 *marks*]

(b) Find the vertex and focus of this conic. Hence, sketch the conic. [4 *marks*]

6. The position vectors of points P, Q and R relative to the origin, O are $\begin{pmatrix} 1 \\ 3 \\ -2 \end{pmatrix}$, $\begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 4 \\ k \\ 5 \end{pmatrix}$ respectively. It is given that the vector product of $\overrightarrow{PQ}$ and $\overrightarrow{PR}$ is $\begin{pmatrix} 12 \\ h \\ -4 \end{pmatrix}$.

(a) Show that $k = -1$ and $h = 2$. [3 *marks*]
(b) Find the area of triangle PQR. [2 *marks*]

Section B [15 *marks*]

Answer **one** *question only.*

You may answer all the questions **but,** *only the first answer will be marked.*

7. Express $f(r) = \dfrac{2}{r^2 + 4r + 3}$ as partial fractions in the form $\dfrac{A}{r+1} + \dfrac{B}{r+3}$. [2 *marks*]

(a) Expand $f(r)$ as a series in ascending powers of r up to the term in r^3 if r is so small that r^4 and higher powers may be neglected
Determine the set of values of r for which the expansion is valid. [5 *marks*]

(b) Given that the r^{th} term, u_r of the series $\dfrac{2}{2(4)} + \dfrac{2}{3(5)} + \dfrac{2}{4(6)} + \ldots$ is $\dfrac{2}{(r+1)(r+3)}$.

(i) Use method of difference, find $\sum_{r=1}^{n} u_r$ in terms of n and deduce $\sum_{r=1}^{\infty} u_r$. [5 *marks*]

(ii) Find the sum of the series $\dfrac{2}{12(14)} + \dfrac{2}{13(15)} + \dfrac{2}{14(16)} + \ldots + \dfrac{2}{21(23)}$ as a fraction. [3 marks]

8. Two lines l_1 and l_2 have equations $\boldsymbol{r} = \begin{pmatrix} 4 \\ -3 \\ 1 \end{pmatrix} + s\begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}$ and $\boldsymbol{r} = \begin{pmatrix} 7 \\ 1 \\ 4 \end{pmatrix} + t\begin{pmatrix} 3 \\ p \\ -1 \end{pmatrix}$ respectively where s and t are real parameters. Given that the lines l_1 and l_2 are perpendicular at point A.

(a) Show that $p = 2$. [2 *marks*]

(b) Determine the coordinates of point A. [3 *marks*]

(c) Show that the Cartesian equation of the plane, p_1 which contains the lines l_1 and l_2 is $5x - 6y + 3z = 41$. [3 *marks*]

(d) It is given the plane, p_2 has equation $3x - y + 2z = 11$.

(i) Calculate the angle between the line l_1 and the plane p_2. [4 *marks*]

(ii) Show that the point $(4,-5,-3)$ lies on both planes p_1 and p_2.
Hence, find the line of intersection between two planes p_1 and p_2 in the form $\boldsymbol{r} = \boldsymbol{a} + \lambda \boldsymbol{b}$ where λ is a real parameter. [3 *marks*]

Section A [45 *marks*]

Answer* all *questions

1 Express $\cos x + \sin x$ in the form $r\cos(x - \alpha)$, where $r > 0$ and $0 < \alpha < \frac{1}{2}\pi$
State the maximum and minimum of $\cos x + \sin x$ [4 *marks*]

(a) Sketch the graph of $y = |\cos x + \sin x|$ for $0 \leq x \leq 2\pi$ [2 *marks*]

(b) Solve the equation $|\cos x + \sin x| = 1$ for $0 \leq x \leq 2\pi$
Hence, use your graph in (a) to find the set of values of x in this interval satisfying the inequality $|\cos x + \sin x| > 1$. [4 *marks*]

$\cos x + \sin x = r\cos(x - \alpha)$ $= r\cos x\cos\alpha + r\sin x\sin\alpha$ $r\cos\alpha = 1 \quad \ldots(1)$ $r\sin\alpha = 1 \quad \ldots(2)$ $r = \sqrt{2} \quad \alpha = \frac{\pi}{4}$ $\cos x + \sin x = \sqrt{2}\cos(x - \frac{\pi}{4})$ Maximum value $= \sqrt{2}$ Minimum value $= -\sqrt{2}$	M1 A1 A1 A1	Both r and α are correct
(a) $y = \lvert\cos x + \sin x\rvert$ $y = \lvert\sqrt{2}\cos(x - \frac{\pi}{4})\rvert$ $y = \lvert\cos x + \sin x\rvert$ $y = 1$ O $\frac{\pi}{4}$ $\frac{3\pi}{4}$ $\frac{5\pi}{4}$ $\frac{7\pi}{4}$ 2π x	D1 D1	Correct shape of $y = \lvert\sqrt{2}\cos(x - \frac{\pi}{4})\rvert$ All correct with maximum value $\sqrt{2}$

x	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
y	1	1	1	1	1

(b) $\lvert\cos x + \sin x\rvert = 1$ $\lvert\sqrt{2}\cos(x - \frac{\pi}{4})\rvert = 1$ $\sqrt{2}\cos(x - \frac{\pi}{4}) = \pm 1$		

$\cos\left(x-\frac{\pi}{4}\right)=\pm\frac{1}{\sqrt{2}}$ $x-\frac{\pi}{4}=\cos^{-1}\left(\pm\frac{1}{\sqrt{2}}\right)$ $x-\frac{\pi}{4}=\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$ $x=\frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi$	M1 A1	Open modulus to ± 1
$\lvert\cos x+\sin x\rvert > 1$ $y=\lvert\cos x+\sin x\rvert \quad y=1$ Set of values of x is $\{x : 0<x<\frac{\pi}{2} \text{ or } \pi<x<\frac{3\pi}{2}\}$	M1 A1	Draw the line $y=1$

2. A sequence $u_1, u_2, u_3, \ldots$ is defined by the recursive formula $u_{n+1}=1+\frac{1}{2}u_n$ with $u_1=1$.

(a) Write down each of the terms u_2, u_3 and u_4 in the form of $\sum_{r=0}^{n-1}\left(\frac{1}{2}\right)^r$. [3 marks]

(b) Show that the explicit formula for u_n is given by $u_n=2\left[1-\left(\frac{1}{2}\right)^n\right]$.
Deduce $\lim_{n\to\infty} u_n$ and state, with a reason, whether the sequence $u_1, u_2, u_3, \ldots$ is a convergent.

[5 marks]

(a) $u_2=1+\frac{1}{2}u_1$ $=1+\frac{1}{2}(1)$ $=1+\frac{1}{2}$ $u_3=1+\frac{1}{2}u_2$ $=1+\frac{1}{2}\left(1+\frac{1}{2}\right)$ $=1+\frac{1}{2}+\left(\frac{1}{2}\right)^2$ $u_4=1+\frac{1}{2}u_3$ $=1+\frac{1}{2}\left[1+\frac{1}{2}+\left(\frac{1}{2}\right)^2\right]$ $=1+\frac{1}{2}+\left(\frac{1}{2}\right)^2+\left(\frac{1}{2}\right)^3$	A1 A1 A1	

(b) (i) $4-h=0$ and $2k+4\neq 0$ $h=4$ $2k\neq -4$ $h=4$ $k\neq -2$ $h=4$ $k\in \mathbb{R}, k\neq -2$	A1	Both h and k are correct
(ii) $4-h=0$ and $2k+4=0$ $h=4$ $2k=-4$ $h=4$ $k=-2$	A1	Both h and k are correct
(iii) $4-h\neq 0$ and $2k+4\in \mathbb{R}$ $h\neq 4$ $k\in \mathbb{R}$ $h\in \mathbb{R}, h\neq 4$	A1	Both h and k are correct

4. Find the polar form of the complex number $2-i\sqrt{12}$. [3 *marks*]
Using De Moivre's theorem, solve the equation $z^5=2-i\sqrt{12}$ giving your answer in the Cartesian form $x+iy$. [5 *marks*]

Modulus $=4$	B1	
Argument $=-\frac{\pi}{3}$	B1	
$2-i\sqrt{12}=4\left[\cos\left(-\frac{\pi}{3}\right)+i\sin\left(-\frac{\pi}{3}\right)\right]$	A1	
$z^5=2-i\sqrt{12}$ $z^5=4\left[\cos\left(-\frac{\pi}{3}\right)+i\sin\left(-\frac{\pi}{3}\right)\right]$		
$z^5=4\left[\cos\left(-\frac{\pi}{3}+2k\pi\right)+i\sin\left(-\frac{\pi}{3}+2k\pi\right)\right]$ $k=0, 1, 2, 3$ and 4	B1	Add $2k\pi$ to $-\frac{\pi}{3}$ where $k=0, 1, 2, 3, 4$
$z=\left(4\left[\cos\left(-\frac{\pi}{3}+2k\pi\right)+i\sin\left(-\frac{\pi}{3}+2k\pi\right)\right]\right)^{\frac{1}{5}}$ $z=4^{\frac{1}{5}}\left[\cos\frac{1}{5}\left(-\frac{\pi}{3}+2k\pi\right)+i\sin\frac{1}{5}\left(-\frac{\pi}{3}+2k\pi\right)\right]$	M1	Apply De Moivre's Theorem on the polar form
$k=0,\ z=4^{\frac{1}{5}}\left[\cos\left(-\frac{\pi}{15}\right)+i\sin\left(-\frac{\pi}{15}\right)\right]$ $=1.2907-0.2743i$	A1	Accept 3, 4 o 5 decimal places.
$k=1,\ z=4^{\frac{1}{5}}\left[\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right]$ $=0.6598+1.1427i$	A1	3 marks for all correct 2 marks for 3-4 correct
$k=2,\ z=4^{\frac{1}{5}}\left[\cos\frac{11\pi}{15}+i\sin\frac{11\pi}{15}\right]$ $=-0.8829+0.9806i$	A1	1 mark for 1-2 correct
$k=3,\ z=4^{\frac{1}{5}}\left[\cos\frac{17\pi}{15}+i\sin\frac{17\pi}{15}\right]$ $=-1.2054-0.5367i$		

(8)

6. The position vectors of points P, Q and R relative to the origin, O are $\begin{pmatrix}1\\3\\-2\end{pmatrix}$, $\begin{pmatrix}2\\3\\1\end{pmatrix}$ and $\begin{pmatrix}4\\k\\5\end{pmatrix}$ respectively. It is given that the vector product of $\overrightarrow{PQ}$ and $\overrightarrow{PR}$ is $\begin{pmatrix}12\\h\\-4\end{pmatrix}$.

(a) Show that $k = -1$ and $h = 2$. [3 marks]

(b) Find the area of triangle PQR. [2 marks]

(a) $\overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP}$

$= \begin{pmatrix}2\\3\\1\end{pmatrix} - \begin{pmatrix}1\\3\\-2\end{pmatrix}$

$= \begin{pmatrix}1\\0\\3\end{pmatrix}$

$\overrightarrow{PR} = \overrightarrow{OR} - \overrightarrow{OP}$

$= \begin{pmatrix}4\\k\\5\end{pmatrix} - \begin{pmatrix}1\\3\\-2\end{pmatrix}$

$= \begin{pmatrix}3\\k-3\\7\end{pmatrix}$

B1 — Both $\overrightarrow{PQ}$ and $\overrightarrow{PR}$ are correct

$\overrightarrow{PQ} \times \overrightarrow{PR} = \begin{pmatrix}1\\0\\3\end{pmatrix} \times \begin{pmatrix}3\\k-3\\7\end{pmatrix}$

$= \begin{vmatrix} i & j & k \\ 1 & 0 & 3 \\ 3 & k-3 & 7 \end{vmatrix}$

$= \begin{vmatrix} 0 & 3 \\ k-3 & 7 \end{vmatrix} i - \begin{vmatrix} 1 & 3 \\ 3 & 7 \end{vmatrix} j + \begin{vmatrix} 1 & 0 \\ 3 & k-3 \end{vmatrix} k$

$= [0 - 3(k-3)]\, i - (7-9)\, j + (k-3-0)\, k$

$= -3(k-3)i + 2j + (k-3)k$

$= \begin{pmatrix}-3(k-3)\\2\\k-3\end{pmatrix}$

M1

$\begin{pmatrix}-3(k-3)\\2\\k-3\end{pmatrix} = \begin{pmatrix}12\\h\\-4\end{pmatrix}$

$-3(k-3) = 12$ $\quad h = 2$

$k - 3 = -4$

$k = 1 - 4 + 3$

$k = -1$

A1

TAMAT SAMPEL

Suka sampel ini?

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- Semester 1: kertas percubaan (trial) negeri, subjek yang dipilih + skema
- Blueprint 30 hari Semester 1 (setiap subjek)
- Semester 3: percubaan 2024, 10 subjek, banyak negeri, soalan + skema dalam satu fail

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