2021-1-PEN-CLB

Section A [45 *marks*]

Answer **all** *questions.*

1. Given that $\mathrm{f}(x) = x^3 + ax^2 + bx - 24$, and $(x+2)^2$ is a factor of $\mathrm{f}(\mathrm{x})$.
   Find the values of a and b. [3]
   Hence, solve the inequality $x^3 - 2x^2 - 22x - 12 > 0$. [4]

2. Expand $(2-x)^{-1}$ as a series in ascending powers of $\frac{1}{x}$, stating the first our terms an the range of values of x for which the expanding is valid. [6]

3. A system of linear equations is given by
$$2x + y - z = 2$$
$$3x + 2y - 3z = 2$$
$$7x + 4y + kz = 6$$
   where k is a real numbers. Write its augmented matrix. [1]

   Find the value of k if the system of linear equations has no unique solutions. [2]

   With this value of k, reduce the augmented matrix for the system to row-echelon form. [3]

   Suggest a general solution for the system of linear equation. [2]

4. The complex numbers z and w are such that $|z| = 2$, $|w| = 12$, $\arg(z) = -\frac{\pi}{4}$ and arg $(w) = \frac{\pi}{6}$.

   (a) Find the modulus and argument of $\frac{w}{z^2}$. [4]

   (b) Find the value of a if $\frac{w}{z^2} + a + bi$ is totally imaginary, where a and b are real numbers. [3]

   (c) Use the De Moivre's theorem, evaluate $\left(\frac{w}{z^2}\right)^6$. [4]

5. Given that P is a moving point such that the sum of the distances from two fixed point $A(0,4)$ and $B(0,12)$ is 10 units.

   State the shape of the locus of P as P moves, and express the equation of the locus in standard form. [4]

   Sketch the locus of P. [3]

6. The equations of two intersecting lines are given by

$$r = 2\mathrm{i} + \mathrm{j} + \lambda(\mathrm{i} + \mathrm{j} + 2\mathrm{k}) \qquad \text{and} \qquad r = 2\mathrm{i} + 2\mathrm{j} - \mathrm{k} + t(\mathrm{i} + 2\mathrm{j} + \mathrm{k})$$

(a) Determine the coordinates of the point of intersection of the two lines. [3]

(b) Find the angles between the two lines. [3]

Section B [15 *marks*]

Answer **one** *question only*.

You may answer all the questions **but**, *only the first answer will be marked*.

7. Express $\mathrm{f}(r) = \dfrac{1}{4r^2 - 1}$ in partial fractions. [4]

Evaluate $\dfrac{1}{1(3)} + \dfrac{1}{3(5)} + \dfrac{1}{5(7)} + \ldots$ [6]

Simplify $\dfrac{1}{4r^2 - 1} - \dfrac{1}{4r^2}$. [1]

Hence, deduce that $\dfrac{1}{2^2} + \dfrac{1}{4^2} + \dfrac{1}{6^2} + \ldots + \dfrac{1}{1000^2} < \dfrac{1}{2}$. [4]

8. Two planes π_1 and π_2 have vector equations $\mathrm{r} \bullet (5\mathrm{i} - 3\mathrm{j} + \mathrm{k}) = 2$ and $\mathrm{r} \bullet (-\mathrm{i} - 9\mathrm{j} + 3\mathrm{k}) = 6$ respectively.

(a) Find the unit vector which is parallel to both planes. [4]

Hence, find a Cartesian equation for the line of intersection, ℓ, of the two planes. [3]

(b) Find the angle between these two planes. [3]

(c) The planes π_3 has equation $\mathrm{r} \bullet (-\mathrm{i} + a\mathrm{j} + b\mathrm{k}) = 6$, where a and b are real numbers. Find the values of a and b given that the 3 planes π_1, π_2 and π_3 intersect in the common line ℓ. [5]

2021-1-PEN-CLB-MS

1.

$f(x)=x^3+ax^2+bx-24=(x+2)^2.(x-6)$	B1	Identify the factor $(x-6)$
$=(x^2+4x+4)(x-6)$ $=x^3-2x^2-20x-24$	M1	Complete expansion
$\therefore a=-2 \quad , b=-20$	A1	CAO
Let $g(x)=x^3-2x^2-22x-12$ $g(x)-f(x)=-2x+12$ $g(x)=(x+2)^2.(x-6)-2(x-6)$	M1	Common factor $(x-6)$
$g(x)=(x-6)((x+2)^2-2)$ $g(x)=(x-6)(x+2-\sqrt{2})(x+2+\sqrt{2})$	M1	Conjugate factors
For $g(x)>0$:	D1	Cubic curve with conjugate roots
$\therefore -2-\sqrt{2}<x<-2+\sqrt{2}$ or $x>6$	A1	Must have 'or'

2.

$(2-x)^{-1}=\left[x\left(\frac{2}{x}-1\right)\right]^{-1}$ $=-\frac{1}{x}\left[1+\left(-\frac{2}{x}\right)\right]^{-1}$	M1	Expression in $\frac{1}{x}$
$=-\frac{1}{x}[1+(-1)]\left(-\frac{2}{x}\right)+\frac{(-1)(-1-1)}{2!}\left(-\frac{2}{x}\right)^2+\frac{(-1)(-1-1)(-1-2)}{3!}\left(-\frac{2}{x}\right)^3+\ldots$	M1	Expansion till four term
$=-\frac{1}{x}\left(1+\frac{2}{x}+\frac{4}{x^2}+\frac{8}{x^3}+\ldots\right)$	A1	Correct expansion
$=-\frac{1}{x}-\frac{2}{x^2}-\frac{4}{x^3}-\frac{8}{x^4}-\ldots$	A1	CAO end with sign (-)
The expansion is valid for $\left\|\frac{2}{x}\right\|<1 \Rightarrow \|x\|>2 \quad \Rightarrow x<-2 \quad \text{or} \quad x>2$	B1 A1	For $\left\|\frac{2}{x}\right\|<1$

3.

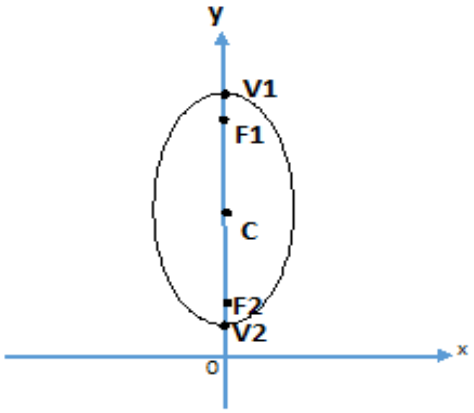
$\left(\begin{array}{ccc\|c} 2 & 1 & -1 & 2 \\ 3 & 2 & -3 & 2 \\ 7 & 4 & k & 6 \end{array}\right)$	B1	CAO
No unique solution: Det $\begin{pmatrix} 2 & 1 & -1 \\ 3 & 2 & -3 \\ 7 & 4 & k \end{pmatrix} = 0$ $2\begin{vmatrix} 2 & -3 \\ 4 & k \end{vmatrix} - 1\begin{vmatrix} 3 & -3 \\ 7 & k \end{vmatrix} + (-1)\begin{vmatrix} 3 & 2 \\ 7 & 4 \end{vmatrix} = 0$ $2(2k+12) - 1(3k+21) - 1(12-14) = 0$ $k = -5$	M1 A1	Any correct method.
$\left(\begin{array}{ccc\|c} 2 & 1 & -1 & 2 \\ 3 & 2 & -3 & 2 \\ 7 & 4 & -5 & 6 \end{array}\right) \begin{array}{c} 3R_1 - 2R_2 \to R_2 \\ R_2 - R_1 \to R_1 \\ 2R_1 + R_1 - R_3 \to R_3 \end{array} \left(\begin{array}{ccc\|c} 1 & 1 & -2 & 0 \\ 0 & -1 & 3 & 2 \\ 0 & 0 & 0 & 0 \end{array}\right)$	M1 M1 A1	2 Zeros below $(1,1)$ Zeros below $(2,2)$ $(2,3)$ R_3 all zero.
Let $z = t$: $-y + 3z = 2 \Rightarrow \quad y = 3t - 2$ $x + y - 2z = 0 \quad \Rightarrow \quad x = 2 - t$	M1 A1	Correct on 'his' CAO or equivalent

4.

(a) $\left\|\frac{w}{z^2}\right\| = \frac{12}{2^2} = 3$	M1 A1	Any equivalent method CAO
$\arg\left(\frac{w}{z^2}\right) = \frac{\pi}{6} - 2\left(-\frac{\pi}{4}\right) = \frac{2\pi}{3}$	M1 A1	Any equivalent method CAO
(b) Let $\frac{w}{z^2} + a + bi = ki$ $3\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right) + a + bi = ki$ $3\cos\frac{2\pi}{3} + a = 0$ $a = \frac{3}{2}$	M1 M1 A1	'his' polar form 'his' real part=0 CAO

$\left(\frac{w}{z^2}\right)^6 = \left[3\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)\right]^6$ $= 3^6\left[\cos 6\left(\frac{2\pi}{3}\right) + i\sin 6\left(\frac{2\pi}{3}\right)\right]$ $= 729(1 + i(0))$ $= 729$	M1 M1 M1 A1	$[\text{his polar form}]^6$ for 3^6 Correct on 'his' CAO

5.

Ellipse	B1	CAO
$2a = 10 \rightarrow a = 5$	B1	CAO
$c = 4$, $b = 3$	B1	CAO (both)
$\frac{(y-8)^2}{25} + \frac{x^2}{9} = 1$	B1	CAO
$V_1(0,13), F_1(0,12), C(0,8), F_2(0,4), V_2(0,3)$  	D1 D1 D1	Shape of ellipse (major axis-y) Label all 5 points All correct.

6.

$\ell_1 : \quad r = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$ $\ell_2 : r = \begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix} + t \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$		

For int. point: $\begin{pmatrix}2+\lambda\\1+\lambda\\2\lambda\end{pmatrix}=\begin{pmatrix}2+t\\2+2t\\-1+t\end{pmatrix}\begin{matrix}2+\lambda=2+t&\to(1)\\\Rightarrow 1+\lambda=2+2t&\to(2)\\2\lambda=-1+t&\to(3)\end{matrix}$	M1	
$(2)-(1): t=-1$ and $\lambda=-1$ From (3): $2(-1)=-1+(-1)\to\therefore$ consistent	M1	Attempt to solve λ and t
$\overrightarrow{OT}=\begin{pmatrix}2\\1\\0\end{pmatrix}+(-1)\begin{pmatrix}1\\1\\2\end{pmatrix}=\begin{pmatrix}1\\0\\-2\end{pmatrix}\quad\therefore T(1,0,-2)$	A1	
$\begin{pmatrix}1\\1\\2\end{pmatrix}\bullet\begin{pmatrix}1\\2\\1\end{pmatrix}=\sqrt{6}.\sqrt{6}\cos\theta$ $1+2+2=\sqrt{6}.\sqrt{6}\cos\theta$ $\cos\theta=\frac{1+2+2}{6}$ $\theta=33.56^\circ$	M1 M1 A1	For $\sqrt{6}$ and Scalar product

7.

$f(r)=\frac{1}{4r^2-1}=\frac{1}{(2r-1)(2r+1)}=\frac{A}{2r-1}+\frac{B}{2r+1}$	B1	CAO
$A=\frac{1}{2}, B=-\frac{1}{2}$	M1 A1	Show working to find either one (A or B) Both (CAO)
$f(r)=\frac{1}{2}\left[\frac{1}{2r-1}-\frac{1}{2r+1}\right]$	A1	CAO
$\frac{1}{1(3)}+\frac{1}{3(5)}+\frac{1}{5(7)}+\ldots=\sum_{r=1}^{\infty}\frac{1}{(2r-1)(2r+1)}$ $=\lim_{n\to\infty}\sum_{r=1}^{n}\frac{1}{2}\left[\frac{1}{2r-1}-\frac{1}{2r+1}\right]$	M1 M1	Sub 'his' partial fractions Using Correct limit notation
$=\lim_{n\to\infty}\left[\frac{1}{2}\left(1+\frac{1}{3}+\frac{1}{5}+\ldots+\frac{1}{2n-1}\right)-\frac{1}{2}\left(\frac{1}{3}+\frac{1}{5}+\ldots+\frac{1}{2n-1}+\frac{1}{2n+1}\right)\right]$	M1	List 'his'enoughterms for difference method
$=\lim_{n\to\infty}\left[\frac{1}{2}(1)-\frac{1}{2}\left(\frac{1}{2n+1}\right)\right]$	M1	'his'correct reduced form

$=\frac{1}{2}-0$	M1	Must show the '0'
$=\frac{1}{2}$	A1	CAO
$\frac{1}{4r^2-1}-\frac{1}{4r^2}=\frac{1}{4r^2\left(4r^2-1\right)}$	M1	
Since $\frac{1}{4r^2\left(4r^2-1\right)}>0\Rightarrow\frac{1}{4r^2-1}>\frac{1}{4r^2}$	A1	Reason & conclusion
$\therefore\sum_{r=1}^{\infty}\frac{1}{4r^2-1}>\sum_{r=1}^{\infty}\frac{1}{4r^2}$	M1	
$\therefore\frac{1}{2}>\sum_{r=1}^{\infty}\frac{1}{\left(2r^2\right)}$	M1	Sub 'his' $\sum_{r=1}^{\infty}\frac{1}{4r^2-1}$
$\therefore\frac{1}{2^2}+\frac{1}{4^2}+\frac{1}{6^2}+\ldots<\frac{1}{2}$ $\therefore\frac{1}{2^2}+\frac{1}{4^2}+\frac{1}{6^2}+\ldots+\frac{1}{1000^2}<\frac{1}{2}$	A1	Reason & conclusion

8.

$\pi_1: r\bullet\begin{pmatrix}5\\-3\\1\end{pmatrix}=2\rightarrow 5x-3y+z=2\quad\rightarrow(1)$ $\pi_2: r\bullet\begin{pmatrix}-1\\-9\\3\end{pmatrix}=6\quad\rightarrow\quad -x-9y+3z=6\quad\rightarrow(2)$		
a) $\begin{pmatrix}5\\-3\\1\end{pmatrix}\text{X}\begin{pmatrix}-1\\-9\\3\end{pmatrix}=\begin{vmatrix}i & j & k\\5 & -3 & 1\\-1 & -9 & 3\end{vmatrix}$ $=\begin{pmatrix}-9+9\\-(15+1)\\-45-3\end{pmatrix}=\begin{pmatrix}0\\-16\\-48\end{pmatrix}=-16\begin{pmatrix}0\\1\\3\end{pmatrix}.$	M1 M1 A1	For $n_1\text{X}\,n_2$ Attempt to apply Cross product

Suka sampel ini?

Kertas Percubaan STPM

Sem 1 · Mathematics T

- Semester 1: kertas percubaan (trial) negeri, subjek yang dipilih + skema
- Blueprint 30 hari Semester 1 (setiap subjek)
- Semester 3: percubaan 2024, 10 subjek, banyak negeri, soalan + skema dalam satu fail

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