2020-1-SAB-KTEKK

Section A [45 marks]

*Answer **all** the questions in this section*

1 The function f and g are defined by $f(x) = 2\ln ln\, x$, $x > 0$ and $g(x) = \sqrt{x-1}$, $x > 1$.

(a) Sketch the graphs of f and g. [2 *marks*]

(b) Find f^{-1} and g^{-1}, stating the domains and ranges of f^{-1} and g^{-1}. [4 *marks*]

(c) Determine whether $g \circ f$ exist. [2 *marks*]

2 The sum of the first four terms is 8 and the sum of the first six term is 24. If the sum of the first n terms is given by $S_n = pn^2 + qn$, determine

(a) the values of p and q. [3 *marks*]

(b) the n^{th} term, U_n. [3 *marks*]

3 Given that $P = (2\ 0\ -1\ 0\ 2\ 1\ -1\ 1\ 2)$ and $Q = (15\ -1\ -4\ -1\ 15\ 4\ -4\ 4\ 16)$.

(a) Show that matrix ***P*** is a non-singular matrix. [2 *marks*]

(b) Determine the matrix $P(Q - 6P)$. Hence deduce the inverse of matrix ***P***. [4 *marks*]

4 Express the complex number $w = 2 - \sqrt{2}i$ in polar form. [3 *marks*]

(a) Find w^5. [2 *marks*]

(b) Solve the equation $w^3 = 2 - \sqrt{2}i$, giving the roots in Cartesian form. [4 *marks*]

5 The equation of a parabola is $8y - 6x = 7 - x^2$.

(a) Obtain the standard form for the equation of the parabola. [2 *marks*]

(b) Hence, find (i) the vertex, V
(ii) the focus, F
(iii) the directrix [3 *marks*]
of the parabola.

(c) Sketch the parabola and indicate the point V, F and the directrix. [3 *marks*]

6 A straight line is defined by $\frac{x-1}{3} = \frac{y+2}{4} = 3 - z$ and a plane is defined by $2x - 3y + 4z = 40$.

(a) The straight line intersects the plane at a point *P*, find the coordinates of *P*. [4 *marks*]

(b) Determine the angle between the straight line and the plane. [3 *marks*]

Section B [15 marks]

*Answer **one** question only in this section*

7 Expand $(1+x)^{-\frac{1}{2}}$ in ascending powers of x up to the term in x^3. [3 *marks*]

Hence, expand $\frac{2+5x}{\sqrt{1+x}}$ in ascending powers of x up to the term in x^3. [5 *marks*]

By substituting $x = 0.08$, show that the value of $\frac{2+5x}{\sqrt{1+x}}$ is $\frac{4}{3}\sqrt{3}$. [4 *marks*]

Use your expansion to estimate the value of $\sqrt{3}$ correct to three decimal places. [3 *marks*]

8

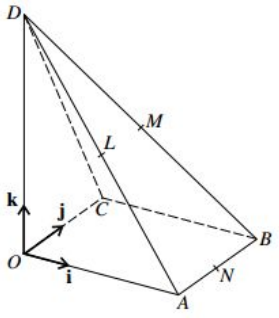


Figure 1

Figure 1 shows a pyramid DOABC. Taking unit vectors **i**, **j**, **k** as shown, the position vectors of A, B, C, D are given by $\vec{OA} = 4i$, $\vec{OB} = 4i + 2j$, $\vec{OC} = 2j$, $\vec{OD} = 6k$

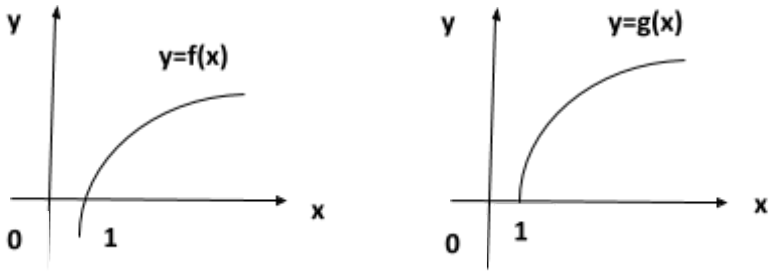
The mid-points of AD, BD and AB are L, M and N respectively.

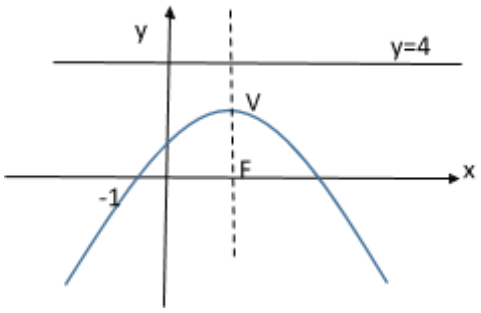
(a) Find the vector $\vec{MN}$ and, by considering the scalar product, the angle between the directions of $\vec{MN}$ and $\vec{OB}$. [8 *marks*]

(b) The point P lying on OD has position vector $p\mathbf{k}$. Determine the value of p for which the line through P and B intersects the line through C and L.

[7 *marks*]

2020-1-SAB-KTEKK-Marking Scheme

	Answers	Marks
1 (a)		D1 D1
(b)	Let $y = f^{-1}(x)$ $f(y) = x$ $2 \ln ln\, y = x$ $f^{-1}(x) = e^{\frac{x}{2}}$ $D_{f^{-1}} = \{x : x \in R\}$; $R_{f^{-1}} = \{y : y \in R,\ y > 1\}$	M1 A1
	Let $y = g^{-1}(x)$ $g(y) = x$ $\sqrt{y-1} = x$ $g^{-1}(x) = x^2 + 1$ $D_{g^{-1}} = \{x : x \in R,\ x > 0\}$; $R_{g^{-1}} = \{y : y \in R,\ y > 1\}$	M1 A1
(c)	$R_f = \{y : y \in R\}$; $D_g = \{x : x > 1\}$ Since $R_f \not\subseteq D_g$, $g \circ f$ does not exist	M1 A1
		8 marks
2(a)	$S_4 = 8$ $\qquad$ $S_6 = 24$ $16p + 4q = 8$ -------- (1) $\qquad$ $36p + 6q = 24$ ---------- (2)	M1 A1
	Solve (1) and (2): $\quad p = 1; \quad q = -2$	A1 (Both)
(b)	$S_n = n^2 - 2n$ $U_n = S_n - S_{n-1}$ $= n^2 - 2n - \left[(n-1)^2 - 2(n-1)\right]$ $= 2n - 3$	M1 A1 A1
		6 marks
3 (a)	Det $P = 2(4-1) - 0 - 1(0+2) = 4$ Since Det $P \neq 0$, then $\boldsymbol{P}$ is a non-singular matrix.	M1 A1
(b)	$P(Q - 6P) = (2\ 0\ -1\ 0\ 2\ 1\ -1\ 1\ 2)[(15\ -1\ -4\ -1\ 15\ 4\ -4\ 4\ 16) - 6(2\ 0\ -1\ 0$ $= (2\ 0\ -1\ 0\ 2\ 1\ -1\ 1\ 2)(3\ -1\ 2\ -1\ 3\ -2\ 2\ -2\ 4)$ $= (4\ 0\ 0\ 0\ 4\ 0\ 0\ 0\ 4) = 4I$	B1 (for $\boldsymbol{Q}$-6$\boldsymbol{P}$) M1 A1
	$P(Q - 6P) = 4I$ $P^{-1} = \frac{1}{4}(Q - 6P) = \frac{1}{4}(3\ -1\ 2\ -1\ 3\ -2\ 2\ -2\ 4)$ $= \left(\frac{3}{4}\ -\frac{1}{4}\ \frac{1}{2}\ -\frac{1}{4}\ \frac{3}{4}\ -\frac{1}{2}\ \frac{1}{2}\ -\frac{1}{2}\ 1\right)$	M1 A1
		7 marks

	Answers	Marks
4 (a)	$w = 2 - \sqrt{2}i$ $\|w\| = \sqrt{6}$ $\arg arg\ (w) = -\frac{\pi}{4}$ $w = \sqrt{6}\left(\cos cos\ \frac{\pi}{4} - isin\frac{\pi}{4}\right)$	B1 B1 B1
(b)	$w^5 = \left(\sqrt{6}\right)^5\left(\cos cos\ \frac{\pi}{4} - isin\frac{\pi}{4}\right)^5$ $= 36\sqrt{6}\left(\cos cos\ \frac{5\pi}{4} - \frac{5\pi}{4}\right)$ $= 36\sqrt{6}\left(-\frac{1}{\sqrt{2}} - i\frac{1}{\sqrt{2}}\right) = -36\sqrt{3} - 36\sqrt{3}i$	B1 B1
(c)	$w^3 = 2 - \sqrt{2}i$ $= \sqrt{6}\left(\cos cos\ \frac{\pi}{4} - isin\frac{\pi}{4}\right)$ $w = \left[\sqrt{6}\left(\cos cos\ \frac{\pi}{4} - isin\frac{\pi}{4}\right)\right]^{\frac{1}{3}}$ $= 6^{\frac{1}{6}}\left[\cos cos\ \frac{2k\pi+\frac{\pi}{4}}{3} - isin\frac{2k\pi+\frac{\pi}{4}}{3}\right]$ for $k = 0$, $w = 6^{\frac{1}{6}}\left(\cos cos\ \frac{\pi}{12} - isin\frac{\pi}{12}\right) = 1.302 - 0.349i$ for $k = 1$, $w = 6^{\frac{1}{6}}\left(\cos cos\ \frac{3\pi}{4} - isin\frac{3\pi}{4}\right) = -0.953 - 0.953i$ for $k = 2$, $w = 6^{\frac{1}{6}}\left(\cos cos\ \frac{17\pi}{12} - isin\frac{17\pi}{12}\right) = -0.349 + 1.302i$	M1 A1 A1 A1
		9 marks
5(a)	$8y - 6x = 7 - x^2$ $x^2 - 6x = -8y + 7$ $(x-3)^2 = -8y + 7 + 9$ $(x-3)^2 = -8(y-2) = 4(-2)(y-2)$	M1 A1
(b) (i)	$V = (3,\ 2)$	B1
(ii)	$F = (3,\ 0)$	B1
(iii)	Directrix, $y = 4$	B1
(c)		D1 D1 D1
		8 marks
6 (a)	From $\frac{x-1}{3} = \frac{y+2}{4} = 3 - z$ Equation is $(x\ y\ z) = (1\ \ -2\ 3) + t(3\ 4\ \ -1)$ $x = 1 + 3t$ $y = -2 + 4t$ $z = 3 - t$ substitute into $2x - 3y + 4z = 40$ $2(1+3t) - 3(-2+4t) + 4(3-t) = 40$ Solve to get $t = -2$, substitute to get $(-5,\ -10,\ 5)$	B1 M1 A1 $(t = -2)$ A1 (Asnwer)
(b)	$\sin sin\ \theta = \frac{(3\ 4\ -1)\bullet(2\ -3\ 4)}{(\sqrt{26})(\sqrt{29})} = \frac{-10}{\sqrt{754}}$ for acute angle, $\sin sin\ \theta = \frac{10}{\sqrt{754}} = 21.4^0$	B1 (for b or n) M1

		A1
		7 marks

	Answers	**Marks**
7(a)	$(1+x)^{-\frac{1}{2}} = 1 + \left(-\frac{1}{2}\right)x + \frac{-\frac{1}{2}\times-\frac{3}{2}}{2!}x^2 + \frac{-\frac{1}{2}\times-\frac{3}{2}\times-\frac{5}{2}}{3!}x^3 + \ldots$ $= 1 - \frac{1}{2}x + \frac{3}{8}x^2 - \frac{5}{16}x^3 + \ldots$	M1 M1 A1
7(b)	$\frac{2+5x}{\sqrt{1+x}} = (2+5x)(1+x)^{-\frac{1}{2}}$ $= (2+5x)(1 - \frac{1}{2}x + \frac{3}{8}x^2 - \frac{5}{16}x^3 + \ldots)$ $= 2 - x + \frac{3}{4}x^2 - \frac{5}{8}x^3 + 5x - \frac{5}{2}x^2 + \frac{15}{8}x^3 + \ldots$ $= 2 + 4x - \frac{7}{4}x^2 + \frac{5}{4}x^3 + \ldots$	B1 M1 M1 A1 A1
7(c)	*by substitute* $x = 0.08$; $\frac{2+5(0.08)}{\sqrt{1+0.08}} = \frac{\frac{12}{5}}{\sqrt{\frac{27}{25}}}$ $= \frac{12}{3\sqrt{3}}$ $= \frac{12}{3\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$ $= \frac{4}{3}\sqrt{3}$	M1 A1 M1 A1
7 (d)	$\frac{2+5x}{\sqrt{1+x}} = \frac{4}{3}\sqrt{3} \approx 2 + 4(0.08) - \frac{7}{4}(0.08)^2 + \frac{5}{4}(0.08)^3$ $\sqrt{3} \approx \frac{3}{4}(2.30944)$ ≈ 1.732 (3 *d.p.*)	M1 M1 A1
		15 marks

	Answers	Marks
8 (a)	$\vec{OM} = \vec{OB} + \vec{BM}$,	
	$\vec{OM} = \vec{OB} + \frac{1}{2}\vec{BD} = 4i + 2j + \frac{1}{2}\left[\vec{BO} + \vec{OD}\right]$	M1
	$= 4i + 2j + \frac{1}{2}[-4i - 2j + 6k]$	
	$= 4i + 2j - 2i - j + 3k$	
	$= 2i + j + 3k$	A1
	$\vec{ON} = \vec{OA} + \vec{AN}$	
	$\vec{ON} = \vec{OA} + \frac{1}{2}\vec{AB}$	
	$= 4i + \frac{1}{2}\left[\vec{AO} + \vec{OB}\right]$	
	$= 4i + \frac{1}{2}[-4i + 4i + 2j]$	
	$= 4i + j$	M1
	$\vec{MN} = \vec{MO} + \vec{ON} = -(2i + j + 3k) + 4i + j$	
	$\vec{MN} = 2i - 3k$	A1
	$\cos\theta = \frac{2(4)}{\sqrt{13}\sqrt{20}} = 0.4961 \quad \theta = 60.26^o$	M1
		A1
		M1 A1
8 (b)	$\vec{PB} = \vec{PO} + \vec{OB}$	M1
	$= -pk + 4i + 2j$	
	$= 4i + 2j - pk$	
	Vector equation of $\vec{PB}$: $r = (4i + 2j) + \alpha(4i + 2j - pk)$	A1
	$r = 4i + 2j + 4\alpha i + 2\alpha j - p\alpha k$	
	$r = (4 + 4\alpha)i + (2 + 2\alpha)j - p\alpha k$	
	$\vec{CL} = \vec{CO} + \vec{OL}$	
	$\vec{CL} = \vec{CO} + \vec{OA} + \vec{AL}$	
	$\vec{CL} = \vec{CO} + \vec{OA} + \frac{1}{2}\vec{AD}$	
	$\vec{CL} = \vec{CO} + \vec{OA} + \frac{1}{2}\left[\vec{AO} + \vec{OD}\right]$	M1
	$\vec{CL} = -2j + 4i + \frac{1}{2}[-4i + 6k]$	
	$\vec{CL} = 2i - 2j + 3k$	
	Vector equation of $\vec{CL}$: $r = 2j + \beta(2i - 2j + 3k)$	
	$r = 2j + 2\beta i - 2\beta j + 3\beta k$	A1
	$r = 2\beta i + (2 - 2\beta)j + 3\beta k$	
	Equate the expressions for r and solve the equation	
	$2 + 2\alpha = \beta$ $\beta = \frac{2}{3}$ $2 + 2\alpha = 2 - 2\beta$ $\alpha = -\beta$	M1
	$-p\alpha = 3\beta$	
	$p\beta = 3\beta$	
	$\therefore p = 3$	B1A1

Suka sampel ini?

Kertas Percubaan STPM

Sem 1 · Mathematics T

- Semester 1: kertas percubaan (trial) negeri, subjek yang dipilih + skema
- Blueprint 30 hari Semester 1 (setiap subjek)
- Semester 3: percubaan 2024, 10 subjek, banyak negeri, soalan + skema dalam satu fail

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