SMK BATU LINTANG KUCHING SMK BATU LINTANG KUCHING SMK BATU LINTANG KUCHING SMK BATU LINTANG KUCHING

954/1 **STPM 2021**

MATHEMATICS (T)

PAPER 1

One and a half hour

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MOCK EXAMINATION 1 (TERM 1)

Section A [45 *marks*]
Answer **all** *questions in this section.*

1. The functions f and g are defined as: $f: x \to 2\ln x\,, x > 0$

$g: x \to \sqrt{x}\,, x \geq 0$

(a) Sketch the graph of f, and give a reason why the inverse function f^{-1} exists. [3 *marks*]

(b) Find f^{-1}, and state its domain. [2 *marks*]

(b) Find the composite function gf^{-1}, and state its range. [3 *marks*]

2. Find the expansion of $\frac{(1+x^2)^{\frac{1}{2}}}{(1-x)^q}$ in ascending powers of x until the term in x^3.

If $q = \frac{1}{2}$, suggest a suitable value of x that enables $\sqrt{\frac{13}{10}}$ to be estimated using the above expansion. Hence, estimate $\sqrt{\frac{13}{10}}$ correct to four decimal places. [8 *marks*]

3. Find the inverse of $\begin{pmatrix} 2 & 2 & 4 \\ 1 & 2 & 3 \\ 1 & 1 & 1 \end{pmatrix}$ using elementary row operations. [4 *marks*]

Hence, solve the following simultaneous equations:

$$x + y + z = 6$$
$$x + y + 2z = 9$$
$$x + 2y + 3z = 13$$

[4 *marks*]

4. Find the roots of $w^4 = -16i$, and sketch the roots on an Argand diagram. [7 *marks*]

5. Show that $y^2 - 4y = x^2 + 2x$ is the equation of a hyperbola and state the centre of this hyperbola. Find the asymptotes of the hyperbola and hence show that they are perpendicular. Sketch the hyperbola. [8 *marks*]

6. Find the equation of the plane which contains the straight line $x - 3 = \frac{4-y}{-3} = \frac{z+1}{2}$ and is perpendicular to the plane $3x + 2y - z = 3$. [6 *marks*]

Section B [15 *marks*]

Answer **one** *question only.*

You may answer all the questions ***but****, only the first answer will be marked.*

7. Express $\cos x + \sin x$ in the form $rcos(x - \alpha)$, where $r > 0$ and $0 < \alpha < \frac{1}{2}\pi$.

Hence, find the minimum and maximum values of $\cos x + \sin x$ and the corresponding values of x in the interval $0 < x < 2\pi$. [7 *marks*]

(a) Sketch the graph of $y = \cos x + \sin x$ for $0 < x < 2\pi$. [3 *marks*]

(b) By drawing appropriate lines on your graph, determine the number of roots in the interval $0 < x < 2\pi$ of each of the following equations.

(i) $\cos x + \sin x = -\frac{1}{2}$ [1 *mark*]

(ii) $\cos x + \sin x = 2$ [1 *mark*]

(c) Find the set of values of x in the interval $0 < x < 2\pi$ for which $|\cos x + \sin x| > 1$. [3 *marks*]

8. The plane π passes through the points with coordinates $P(4, -1, -3)$, $Q(-2, -5, 2)$ and $R(4, -3, -2)$, find a Cartesian equation of the plane π. [4 *marks*]

The line l_1 has equation $\frac{x-1}{2} = \frac{y-2}{-4} = z + 3$ and the line l_2 has equation $x + 2 = \frac{y-1}{5} = \frac{z-3}{k}$ where k is a constant. Given that l_1 and l_2 intersect, find

(a) the value of k. [4 *marks*]

(b) the coordinates of the point at which the l_2 intersects the plane π. [4 *marks*]

(c) the acute angle between the line l_2 and the plane π. [3 *marks*]

~~The roots of education are bitter, but the fruit is sweet.~~
Aristotle

Section A

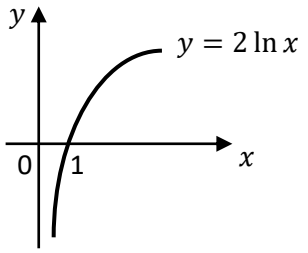
1. **The functions f and g are defined as:** $f: x \to 2\ln x, x > 0$
$g: x \to \sqrt{x}, x \geq 0$

(a) **Sketch the graph of f, and give a reason why the inverse function f^{-1} exists.** **[3 *marks*]**

(b) **Find f^{-1}, and state its domain.** **[2 *marks*]**

(b) **Find the composite function gf^{-1}, and state its range.** **[3 *marks*]**

(a)



D1

Since $f(x)$ is a one-to-one function, $f^{-1}(x)$ exists. **M1 A1**

(b) Let $f^{-1}(x) = y$
$f(y) = x$
$2\ln y = x$
$\ln y = \frac{x}{2}$
$y = e^{\frac{x}{2}}$

$f^{-1}: x \to e^{\frac{x}{2}}, \ x \in \mathbb{R}$ **A1**

Domain of $f^{-1}(x) = \{x: x \in \mathbb{R}\}$ **A1**

(c) $gf^{-1}(x) = \sqrt{e^{\frac{x}{2}}} = e^{\frac{x}{4}}$ **M1**

$gf^{-1}: x \to e^{\frac{x}{4}}, \ x \in \mathbb{R}$ **A1**

Range of $gf^{-1}(x) = \{y: y > 0\}$ **A1**

2. **Find the expansion of $\frac{(1+x^2)^{\frac{1}{2}}}{(1-x)^q}$ in ascending powers of x until the term in x^3.**

If $q=\frac{1}{2}$, suggest a suitable value of x that enables $\sqrt{\frac{13}{10}}$ to be estimated using the above expansion. Hence, estimate $\sqrt{\frac{13}{10}}$ correct to four decimal places. **[8 *marks*]**

$\frac{(1+x^2)^{\frac{1}{2}}}{(1-x)^q} = (1+x^2)^{\frac{1}{2}}(1-x)^{-q}$ **M1**

$= \left[1+\frac{1}{2}x^2+\cdots\right]\left[1+(-q)(-x)+\frac{(-q)(-q-1)}{2!}(-x)^2+\frac{(-q)(-q-1)(-q-2)}{3!}(-x)^3+\cdots\right]$ **M1 M1**

$= \left[1+\frac{1}{2}x^2+\cdots\right]\left[1+qx+\frac{(q)(q+1)}{2}x^2+\frac{(q)(q+1)(q+2)}{6}x^3+\cdots\right]$

$= 1+qx+\frac{(q)(q+1)}{2}x^2+\frac{(q)(q+1)(q+2)}{6}x^3$
$\quad +\frac{1}{2}x^2 \quad +\frac{1}{2}qx^3$

$= 1+qx+\left(\frac{1}{2}+\frac{(q)(q+1)}{2}\right)x^2+\left(\frac{1}{2}q+\frac{(q)(q+1)(q+2)}{6}\right)x^3+\cdots$

$= 1+qx+\left(\frac{1+(q)(q+1)}{2}\right)x^2+\left(\frac{3q+(q)(q+1)(q+2)}{6}\right)x^3+\cdots$ expansion valid if $0<x<1$ **A1**

Given $q=\frac{1}{2}$, $\sqrt{\frac{(1+x^2)}{(1-x)}} = \sqrt{\frac{13}{10}}$ **M1**

$10x^2+13x-3=0$

$(2x+3)(5x-1)=0$

$x = -\frac{3}{2}, \frac{1}{5}$

Let $x=\frac{1}{5}$, **B1**

$\sqrt{\frac{\left(1+\left(\frac{1}{5}\right)^2\right)}{\left(1-\frac{1}{5}\right)}} \approx 1+\frac{1}{2}\left(\frac{1}{5}\right)+\left(\frac{1+\left(\frac{1}{2}\right)\left(\frac{1}{2}+1\right)}{2}\right)\left(\frac{1}{5}\right)^2+\left(\frac{3\left(\frac{1}{2}\right)+\left(\frac{1}{2}\right)\left(\frac{1}{2}+1\right)\left(\frac{1}{2}+2\right)}{6}\right)\left(\frac{1}{5}\right)^3+\cdots$ **M1**

$\sqrt{\frac{13}{10}} \approx 1.1395$ (4 dp) **A1**

3. **Find the inverse of** $\begin{pmatrix} 2 & 2 & 4 \\ 1 & 2 & 3 \\ 1 & 1 & 1 \end{pmatrix}$ **using elementary row operations.** **[4 *marks*]**

Hence, solve the following simultaneous equations:

$$x + y + z = 6$$
$$x + y + 2z = 9$$
$$x + 2y + 3z = 13$$

[4 *marks*]

$$\left(\begin{array}{ccc|ccc} 2 & 2 & 4 & 1 & 0 & 0 \\ 1 & 2 & 3 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 \end{array}\right) \quad \textbf{M1}$$

$$\xrightarrow{R_3 \leftrightarrow R_1} \left(\begin{array}{ccc|ccc} 1 & 1 & 1 & 0 & 0 & 1 \\ 1 & 2 & 3 & 0 & 1 & 0 \\ 2 & 2 & 4 & 1 & 0 & 0 \end{array}\right)$$

$$\xrightarrow{\substack{R_2 - R_1 \to R_2 \\ R_3 - 2R_1 \to R_3}} \left(\begin{array}{ccc|ccc} 1 & 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 2 & 0 & 1 & -1 \\ 0 & 0 & 2 & 1 & 0 & -2 \end{array}\right) \quad \textbf{M1}$$

$$\xrightarrow{\frac{1}{2}R_3 \to R_3} \left(\begin{array}{ccc|ccc} 1 & 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 2 & 0 & 1 & -1 \\ 0 & 0 & 1 & \frac{1}{2} & 0 & -1 \end{array}\right)$$

$$\xrightarrow{R_2 - 2R_3 \to R_2} \left(\begin{array}{ccc|ccc} 1 & 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 & 1 & 1 \\ 0 & 0 & 1 & \frac{1}{2} & 0 & -1 \end{array}\right)$$

$$\xrightarrow{R_1 - R_3 \to R_1} \left(\begin{array}{ccc|ccc} 1 & 1 & 0 & -\frac{1}{2} & 0 & 2 \\ 0 & 1 & 0 & -1 & 1 & 1 \\ 0 & 0 & 1 & \frac{1}{2} & 0 & -1 \end{array}\right)$$

$$\xrightarrow{R_1 - R_2 \to R_1} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -\frac{1}{2} & -1 & 1 \\ 0 & 1 & 0 & -1 & 1 & 1 \\ 0 & 0 & 1 & \frac{1}{2} & 0 & -1 \end{array}\right) \quad \textbf{M1}$$

$$\begin{pmatrix} 2 & 2 & 4 \\ 1 & 2 & 3 \\ 1 & 1 & 1 \end{pmatrix}^{-1} = \begin{pmatrix} -\frac{1}{2} & -1 & 1 \\ -1 & 1 & 1 \\ \frac{1}{2} & 0 & -1 \end{pmatrix} \quad \textbf{A1}$$

$x + y + z = 6$
$x + y + 2z = 9$ $\qquad 2x + 2y + 4z = 18$ **B1**
$x + 2y + 3z = 13$ $\qquad x + 2y + 3z = 13$
$\qquad\qquad\qquad\quad x + y + z = 6$

$$\begin{pmatrix} 2 & 2 & 4 \\ 1 & 2 & 3 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 18 \\ 13 \\ 6 \end{pmatrix} \quad \textbf{M1}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -\frac{1}{2} & -1 & 1 \\ -1 & 1 & 1 \\ \frac{1}{2} & 0 & -1 \end{pmatrix} \begin{pmatrix} 18 \\ 13 \\ 6 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix} \quad \textbf{M1}$$

$x = 2, y = 1, z = 3$ **A1**

4. **Find the roots of $w^4 = -16i$, and sketch the roots on an Argand diagram.** **[7 *marks*]**

$$w^4 = 16\left[\cos\left(-\frac{\pi}{2}\right) + i\sin\left(-\frac{\pi}{2}\right)\right] \quad \textbf{B1}$$

$$w = \left[16\left[\cos\left(2k\pi - \frac{\pi}{2}\right) + i\sin\left(2k\pi - \frac{\pi}{2}\right)\right]\right]^{\frac{1}{4}}, k = 0, 1, 2, 3 \quad \textbf{M1}$$

$$w = 2\left[\cos\left(\frac{4k-1}{8}\right)\pi + i\sin\left(\frac{4k-1}{8}\right)\pi\right], k = 0, 1, 2, 3 \quad \textbf{M1}$$

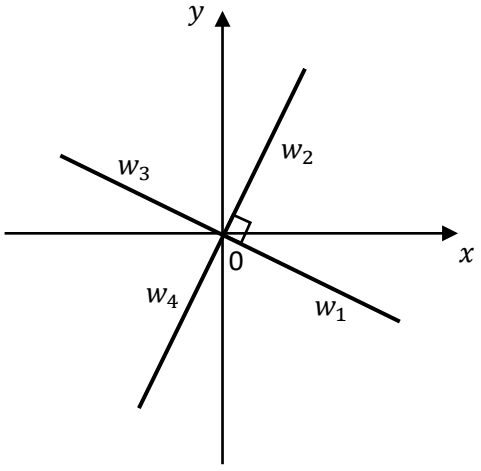
$k = 0,\quad w_1 = 2\left[\cos\left(-\frac{1}{8}\pi\right) + i\sin\left(-\frac{1}{8}\pi\right)\right]$
$= 1.848 - 0.7654i$

$k = 1,\quad w_2 = 2\left[\cos\left(\frac{3}{8}\pi\right) + i\sin\left(\frac{3}{8}\pi\right)\right]$
$= 0.7654 + 1.848i$

$k = 2,\quad w_3 = 2\left[\cos\left(\frac{7}{8}\pi\right) + i\sin\left(\frac{7}{8}\pi\right)\right]$
$= -1.848 + 0.7654i$

$k = 3,\quad w_4 = 2\left[\cos\left(\frac{11}{8}\pi\right) + i\sin\left(\frac{11}{8}\pi\right)\right]$
$= 2\left[\cos\left(-\frac{5}{8}\pi\right) + i\sin\left(-\frac{5}{8}\pi\right)\right]$
$= -0.7654 - 1.848i$

M1 A1



D1 - $w_1 - w_4$ seen

D1 - perpendicular

5. Show that $y^2 - 4y = x^2 + 2x$ is the equation of a hyperbola and state the centre of this hyperbola. Find the asymptotes of the hyperbola and hence show that they are perpendicular. Sketch the hyperbola. [8 *marks*]

$y^2 - 4y = x^2 + 2x$
$y^2 - 4y + (-2)^2 - (-2)^2 = x^2 + 2x + 1^2 - 1^2$ **M1**
$(y-2)^2 - 4 = (x+1)^2 - 1$
$(y-2)^2 - (x+1)^2 = 3$
$\frac{(y-2)^2}{3} - \frac{(x+1)^2}{3} = 1$ **A1**

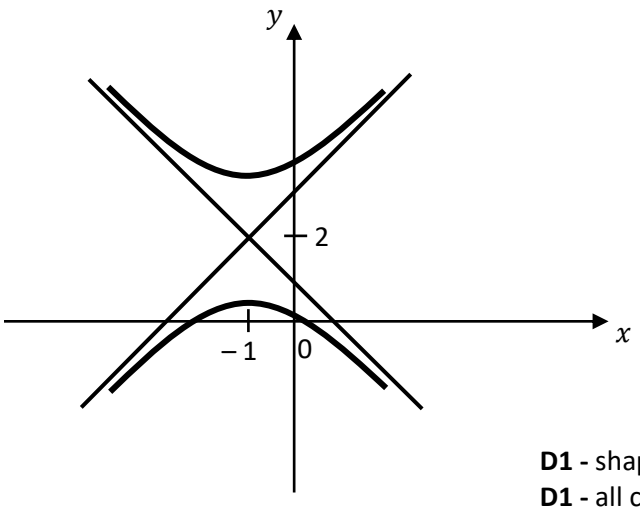
The equation is a hyperbola with centre $(-1, 2)$ **A1**

Equations of asymptotes:

$y - 2 = \pm(x+1)$ **M1**

$y - 2 = x + 1$	$y - 2 = -(x+1)$
$y = x + 3$	$y = -x + 1$
$m_1 = 1$	$m_2 = -1$

$m_1 \times m_2 = 1 \times -1 = -1$ **M1**
The asymptotes are perpendicular. **A1**



D1 - shape & passes (0, 0)
D1 - all correct

6. **Find the equation of the plane which contains the straight line $x - 3 = \frac{4-y}{-3} = \frac{z+1}{2}$ and is perpendicular to the plane $3x + 2y - z = 3$.** **[6 *marks*]**

line: $\boldsymbol{r} = \begin{pmatrix} 3 \\ 4 \\ -1 \end{pmatrix} + t\begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix}$

M1

plane: $\boldsymbol{r} \cdot \begin{pmatrix} 3 \\ 2 \\ -1 \end{pmatrix} = 3$

$\boldsymbol{n} = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ 3 & 2 & -1 \\ 1 & 3 & 2 \end{vmatrix}$ **M1**

$= (4 + 3)\boldsymbol{i} - (6 + 1)\boldsymbol{j} + (9 - 2)\boldsymbol{k}$ **M1**

$= 7\boldsymbol{i} - 7\boldsymbol{j} + 7\boldsymbol{k}$

$= 7(\boldsymbol{i} - \boldsymbol{j} + \boldsymbol{k})$

$\boldsymbol{r} \cdot \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$ **M1**

$\boldsymbol{r} \cdot \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} = -2$ **M1**

Equation of the plane is $x - y + z = -2$ **A1 -** accept Cartesian form only

Suka sampel ini?

Kertas Percubaan STPM

Sem 1 · Mathematics T

- Semester 1: kertas percubaan (trial) negeri, subjek yang dipilih + skema
- Blueprint 30 hari Semester 1 (setiap subjek)
- Semester 3: percubaan 2024, 10 subjek, banyak negeri, soalan + skema dalam satu fail

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