Section A [45 marks]

Answer **all** questions in this section.

1. The following data shows the marks by 24 students in a particular test.

23	39	52	20	65	88	51	22
48	37	41	25	37	38	59	59
68	46	23	69	42	41	95	40

(a) Construct a stemplot to represent the data. [2]

(b) Determine the median and interquartile range. [4]

(c) Comment on the shape of the distribution. [1]

2. A factory produces microchips using 3 types of machines *S*, *T* and *U*. Machines *S*, *T* and *U* produce 33%, 21% and 46% of the microchips respectively.
The quality control department finds out that the percentages of defective microchips produced by machines *S*, *T* and *U* are 3.6%, 4.3% and 3.1% respectively.

(a) Determine the probability that a microchip selected at random is defective. [2]

(b) If the microchip is defective, calculate the probability that the microchip is produced by

(i) machine *S*,

(ii) machine *T* or machine *U*. [4]

3. The continuous random variable *X* has probability density function

$$f(x) = \begin{cases} ax + b\ , & -1 \le x \le c \\ 0\ , & otherwise \end{cases}$$

with P ($X \le -\frac{1}{2}$) = $\frac{1}{16}$ and P ($X \ge \frac{1}{2}$) = $\frac{7}{16}$.

(a) Calculate the values of a and b and show that c = 1. [5]

(b) Find the cumulative probability distribution. [3]

4. In a random sample of 50 postgraduate students in a university, 36 students are satisfied with the services at the main library of the university.

(a) Determine the smallest sample size needed to estimate the population proportion with an error not exceeding 0.1 at the 95% confidence level. State any assumption made. [6]

(b) State the effect on the sample size

(i) if the error is larger than 0.1 with the confidence level unchanged, [1]

(ii) if the confidence level is higher than 95% with the error unchanged. [1]

5. A report claims that 15 percent of Malaysian teenagers play digital games on average for at least 3 hours every day. Choo is asked by his teacher to investigate his belief that a higher percentage of teenagers play digital games on average for at least 3 hours every day. Choo finds that, of the 12 students in his class, 3 of them report an average of at least 3 hours every day. Test, at the 10 % level of significance, whether the corresponding population percentage for teenagers is higher than 15 percent. [6]

6. Five geese are observed over a period of 140 days and the number of eggs collected each day is recorded and summarized as shown below.

Number of eggs (x)	0	1	2	3	4	5
Number of days (f)	2	10	27	42	39	20

Perform a chi-squared test, using a 10% significance level to test whether the above data fit a binomial model with parameter $n = 5$ and $p = \frac{2}{3}$. [10]

SECTION B [15 MARKS]

*Answer **one** questions only from this section.*

7. Each of a random sample of 50 one ringgit coins was weighed and their masses, x grams , are summarised by $\sum x = 474.51$ $\sum x^2 = 4503.8276$

(a) Use an unbiased of variance to calculate an approximate 90% confidence interval for the mean mass (in grams) of all one-ringgit coins. [8]

(b) Estimate the size of a random sample of one-ringgit coins that would be required to give a 98% confidence interval whose width is half that of the interval calculated in (a) [5]

(c) It was found that the scales were consistently under weighing by 0.05 grams. State whether the result in (a) be should be amended. If yes, give the amended values. [2]

8 (a) The continuous random variable X has cumulative distribution function

$$\mathrm{F}(x) = \begin{cases} 0, & x < 0, \\ \frac{1}{3}x^2(4 - x^2), & 0 \le x \le 1, \\ 1 & x > 1. \end{cases}$$

(i) Find $\mathrm{P}(X > 0.7)$. [2]

(ii) Find the probability density function $\mathrm{f}(x)$ of X. [2]

(iii) Calculate $\mathrm{E}(X)$ and show that, to 3 significant figures, $\mathrm{Var}(X) = 0.0573$. [4]

(b) A manufacturer invented a new device to be attached to the engine of cars to improve fuel consumption. A test is to be carried out to investigate the effect of the device on the rate of fuel consumption of cars. The device was attached to the engines of 100 cars of a particular model in which rate of fuel consumption is known to have a mean of 18 km per litre and standard deviation 1 km per litre.

A statistical test is carried out at 8% level of significance and it is found that the device affects the rate of fuel consumption.

(i) State what you understand by the expression at '8% significance level' in the context of the question. [1]

(ii) Find the range of values of the sample mean. [6]

7. Each of a random sample of 50 one ringgit coins was weighed and their masses, x grams , are summarised by $\sum x = 474.51$ $\sum x^2 = 4503.8276$

(a) Use an unbiased of variance to calculate an approximate 90% confidence interval for the mean mass (in grams) of all one-ringgit coins. [8]

(b) Estimate the size of a random sample of one-ringgit coins that would be required to give a 98% confidence interval whose width is half that of the interval calculated in (a) [5]

(c) It was found that the scales were consistently under weighing by 0.05grams. State whether the result in (a) be should be amended. If yes, give the amended values. [2]

7a	X = mass of coins	
	Variance, $S^2 = \frac{4503.8276}{50} - \left(\frac{474.51}{50}\right)^2$	M1
	Unbiased estimate of variance, $\hat{\sigma}^2 = \frac{50}{49}\left[\frac{4503.8276}{50} - \left(\frac{474.51}{50}\right)^2\right]$	M1
	= 0.012914	A1
	$\bar{x} = \frac{474.51}{50}$	M1
	= 9.4902	A1
	$\alpha = 0.1$, $Z_{\frac{\alpha}{2}} = Z_{0.05} = 1.645$	B1
	90% confidence interval for mean mass $= \left(9.4902 - 1.645\sqrt{\frac{0.012914}{50}}, 9.4902 + 1.645\sqrt{\frac{0.012914}{50}}\right)$	M1
	=(9.4638 , 9.5166)	A1
b	width of 90 % CI $= \frac{9.5166-9.4638}{2}$	M1
	width of 98% confidence interval $= \frac{9.5166-9.4638}{2} = 0.0264$	B1
	when $\alpha = 0.02$, $Z_{\frac{\alpha}{2}} = Z_{0.01} = 2.326$	B1
	If size of random sample = n, $2\left(2.326\sqrt{\frac{0.01291}{n}}\right) = 0.0264$	M1
	n = 400.86 The size of the random sample is 401	A1
c	Result in (a) should be amended to the right 0.05 units.	B1
	The new confidence interval would be (9.5138, 9.5666)	B1

8 (a) The continuous random variable X has cumulative distribution function

$$\mathrm{F}(x)=\begin{cases}0, & x<0,\\ \frac{1}{3}x^2(4-x^2), & 0\le x\le 1,\\ 1 & x>1.\end{cases}$$

(i) Find P(X > 0.7). [2]
(ii) Find the probability density function f(x) of X. [2]
(iii) Calculate E(X) and show that, to 3 significant figures, Var (X) = 0.0573. [4]

(b) A manufacturer invented a new device to be attached to the engine of cars to improve fuel consumption. A test is to be carried out to investigate the effect of the device on the rate of fuel consumption of cars. The device was attached to the engines of 100 cars of a particular model in which rate of fuel consumption is known to have a mean of 18 km per litre and standard deviation 1 km per litre.
A statistical test is carried out at 8% level of significance and it is found that the device affects the rate of fuel consumption.
(i) State what you understand by the expression at '8% significance level' in the context of the question. [1]
(ii) Find the range of values of the sample mean. [6]

8(a) (i)	P(X > 0.7) = 1 - P($X \le 0.7$) = 1 – F(0.7)	M1
	= 0.4267	A1
(ii)	f(x) = $\frac{\mathrm{d}}{\mathrm{d}x}$ F(x) = $\frac{8}{3}x-\frac{4}{3}x^3$	M1
	$f(x)=\begin{cases}\frac{8}{3}x-\frac{4}{3}x^3, & 0\le x\le 1\\ 0, & otherwise\end{cases}$	A1
(iii)	E(X) = $\int_0^1 x\left(\frac{8}{3}x-\frac{4}{3}x^3\right)dx=\int_0^1\left(\frac{8}{3}x^2-\frac{4}{3}x^4\right)dx$	M1
	$\left[\frac{8}{9}x^3-\frac{4}{15}x^5\right]_0^1=\frac{8}{9}(1)^3-\frac{4}{15}(1)^5-0$	
	= $\frac{28}{45}$ or 0.62222	A1
	Var (X) = $\int_0^1\frac{4}{3}(2x^3-x^5)\,\mathrm{d}x-\left(\frac{28}{45}\right)^2$	M1
	= $\left[\frac{4}{3}\left(\frac{2x^4}{4}-\frac{x^6}{6}\right)\right]_0^1-\left(\frac{28}{45}\right)^2$	
	= $\frac{116}{2025}$ = 0.05728 = 0.0573 (shown)	A1

8(b) (i)	' 8% level of significance' means that there is a 0.08 probability that the test shows that the mean rate of fuel consumption is affected (*rejecting H_0*) when it is in fact 18 km per litre(*When H_0 is in fact true*).	B1
(ii)	$H_0 : \mu = 18$ $H_{1:} \mu \neq 18$	B1 (both)
	Under H_0 and by Central Limit Theorem	
	$\bar{X} \sim N\left(18, \frac{1^2}{100}\right)$	B1
	$\mu_0 = 18$, $\sigma = 1$, n =100	
	Level of significance = 8% = 0.08	
	Test statistic z = $\frac{\bar{x} - 18}{\frac{1}{\sqrt{100}}}$	M1
	Critical region: Reject H_0 if $z < -1.751$ or $z > 1.751$	B1
	Device affects the rate of fuel consumption $\Rightarrow$ Reject H_0	
	$\Rightarrow z = \frac{\bar{x} - 18}{\frac{1}{\sqrt{100}}} < -1.751$ or $z = \frac{\bar{x} - 18}{\frac{1}{\sqrt{100}}} > 1.751$	M1
	$\therefore \bar{X} < 17.825$ or $\bar{X} > 18.175$	A1

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- Semester 1: kertas percubaan (trial) negeri, subjek yang dipilih + skema
- Blueprint 30 hari Semester 1 (setiap subjek)
- Semester 3: percubaan 2024, 10 subjek, banyak negeri, soalan + skema dalam satu fail

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